

Choice Based Credit System (CBCS)

UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

UNDERGRADUATE PROGRAMME
(Courses effective from Academic Year 2015-16)



SYLLABUS OF COURSES TO BE OFFERED

Core Courses, Elective Courses & Ability Enhancement Courses

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Undergraduate Programme Secretariat

Preamble

The University Grants Commission (UGC) has initiated several measures to bring equity, efficiency and excellence in the Higher Education System of country. The important measures taken to enhance academic standards and quality in higher education include innovation and improvements in curriculum, teaching-learning process, examination and evaluation systems, besides governance and other matters.

The UGC has formulated various regulations and guidelines from time to time to improve the higher education system and maintain minimum standards and quality across the Higher Educational Institutions (HEIs) in India. The academic reforms recommended by the UGC in the recent past have led to overall improvement in the higher education system. However, due to lot of diversity in the system of higher education, there are multiple approaches followed by universities towards examination, evaluation and grading system. While the HEIs must have the flexibility and freedom in designing the examination and evaluation methods that best fits the curriculum, syllabi and teaching-learning methods, there is a need to devise a sensible system for awarding the grades based on the performance of students. Presently the performance of the students is reported using the conventional system of marks secured in the examinations or grades or both. The conversion from marks to letter grades and the letter grades used vary widely across the HEIs in the country. This creates difficulty for the academia and the employers to understand and infer the performance of the students graduating from different universities and colleges based on grades.

The grading system is considered to be better than the conventional marks system and hence it has been followed in the top institutions in India and abroad. So it is desirable to introduce uniform grading system. This will facilitate student mobility across institutions within and across countries and also enable potential employers to assess the performance of students. To bring in the desired uniformity, in grading system and method for computing the cumulative grade point average (CGPA) based on the performance of students in the examinations, the UGC has formulated these guidelines.

CHOICE BASED CREDIT SYSTEM (CBCS):

The CBCS provides an opportunity for the students to choose courses from the prescribed courses comprising core, elective/minor or skill based courses. The courses can be evaluated following the grading system, which is considered to be better than the conventional marks system. Therefore, it is necessary to introduce uniform grading system in the entire higher education in India. This will benefit the students to move across institutions within India to begin with and across countries. The uniform grading system will also enable potential employers in assessing the performance of the candidates. In order to bring uniformity in evaluation system and computation of the Cumulative Grade Point Average (CGPA) based on student's performance in examinations, the UGC has formulated the guidelines to be followed.

Outline of Choice Based Credit System:

- 1. Core Course:** A course, which should compulsorily be studied by a candidate as a core requirement is termed as a Core course.
- 2. Elective Course:** Generally a course which can be chosen from a pool of courses and which may be very specific or specialized or advanced or supportive to the discipline/ subject of study or which provides an extended scope or which enables an exposure to some other discipline/subject/domain or nurtures the candidate's proficiency/skill is called an Elective Course.
 - 2.1 Discipline Specific Elective (DSE) Course:** Elective courses may be offered by the main discipline/subject of study is referred to as Discipline Specific Elective. The University/Institute may also offer discipline related Elective courses of interdisciplinary nature (to be offered by main discipline/subject of study).
 - 2.2 Dissertation/Project:** An elective course designed to acquire special/advanced knowledge, such as supplement study/support study to a project work, and a candidate studies such a course on his own with an advisory support by a teacher/faculty member is called dissertation/project.
 - 2.3 Generic Elective (GE) Course:** An elective course chosen generally from an unrelated discipline/subject, with an intention to seek exposure is called a Generic Elective.

P.S.: A core course offered in a discipline/subject may be treated as an elective by other discipline/subject and vice versa and such electives may also be referred to as Generic Elective.
- 3. Ability Enhancement Courses (AEC)/Competency Improvement Courses/Skill Development Courses/Foundation Course:** The Ability Enhancement (AE) Courses may be of two kinds: AE Compulsory Course (AECC) and AE Elective Course (AEEC). "AECC" courses are the courses based upon the content that leads to Knowledge enhancement. They ((i) Environmental Science, (ii) English/MIL Communication) are mandatory for all disciplines. AEEC courses are value-based and/or skill-based and are aimed at providing hands-on-training, competencies, skills, etc.
 - 3.1 AE Compulsory Course (AECC):** Environmental Science, English Communication/MIL Communication.
 - 3.2 AE Elective Course (AEEC):** These courses may be chosen from a pool of courses designed to provide value-based and/or skill-based instruction.

Project work/Dissertation is considered as a special course involving application of knowledge in solving / analyzing /exploring a real life situation / difficult problem. A Project/Dissertation work would be of 6 credits. A Project/Dissertation work may be given in lieu of a discipline specific elective paper.

Details of courses under B.A (Honors), B.Com (Honors) & B.Sc. (Honors)

Course	*Credits	
	Theory+ Practical	Theory + Tutorial
<u>I. Core Course</u>		
(14 Papers)	14X4= 56	14X5=70
Core Course Practical / Tutorial*		
(14 Papers)	14X2=28	14X1=14
<u>II. Elective Course</u>		
(8 Papers)		
A.1. Discipline Specific Elective	4X4=16	4X5=20
(4 Papers)		
A.2. Discipline Specific Elective Practical/ Tutorial*	4 X 2=8	4X1=4
(4 Papers)		
B.1. Generic Elective/ Interdisciplinary	4X4=16	4X5=20
(4 Papers)		
B.2. Generic Elective Practical/ Tutorial*	4 X 2=8	4X1=4
(4 Papers)		
• Optional Dissertation or project work in place of one Discipline Specific Elective paper (6 credits) in 6th Semester		
<u>III. Ability Enhancement Courses</u>		
1. Ability Enhancement Compulsory		
(2 Papers of 2 credit each)	2 X 2=4	2 X 2=4
Environmental Science		
English/MIL Communication		
2. Ability Enhancement Elective (Skill Based)		
(Minimum 2)	2 X 2=4	2 X 2=4
(2 Papers of 2 credit each)		
Total credit	140	140
Institute should evolve a system/policy about ECA/ General Interest/Hobby/Sports/NCC/NSS/related courses on its own.		

* wherever there is a practical there will be no tutorial and vice-versa

Structure

	Core Course (14)	Ability Enhancement Compulsory Course (AECC) (2)	Skill Enhancement Course (SEC) (2)	Elective Discipline Specific DSE (4)	Elective: Generic (4)
I	C 1 Calculus (including practicals)	(English communication/MIL) /Environmental Science			GE-1
	C 2 Algebra				
II	C 3 Real Analysis	(English communication/MIL) /Environmental Science			GE-2
	C 4 Differential Equations (including practicals)				
III	C 5 Theory of Real functions		SEC-1 LaTeX and		GE-3

			HTML		
	C 6 Group Theory-I				
	C 7 Multivariate Calculus (including practicals)				
IV	C 8 Partial Differential Equations (including practicals)		SEC-2 Computer Algebra Systems and Related Softwares		GE-4
	C 9 Riemann Integration & Series of functions				
	C 10 Ring Theory & Linear Algebra-I				
V	C 11 Metric Spaces			DSE-1 (including practicals) (i) Numerical Methods or (ii) Mathematical Modeling and Graph Theory or (iii) C++ Programming DSE-2 (i) Mathematical Finance or (ii) Discrete Mathematics	

				<p>or</p> <p>(iii) Cryptography and Network Security</p>	
	C 12 Group Theory-II				
VI	C 13 Complex Analysis (including practicals)			<p>DSE-3</p> <p>(i) Probability theory & Statistics or (ii) Mechanics or (iii) Bio-Mathematics</p> <p>DSE-4</p> <p>(i) Number Theory or (ii) Linear Programming and Theory of Games or (iii) Applications of Algebra</p>	
	C 14 Ring Theory and Linear Algebra-II				

C1- Calculus (including practicals)

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practical (each in group of 15-20)

Hyperbolic functions, Higher order derivatives, Applications of Leibnitz rule.

[2]: Chapter 7 (Section 7.8)

The first derivative test, concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test, limits at infinity, graphs with asymptotes. Graphs with asymptotes, L'Hopital's rule, applications in business, economics and life sciences.

[1]: Chapter 4 (Sections 4.3, 4.4, 4.5, 4.7)

Parametric representation of curves and tracing of parametric curves, Polar coordinates and tracing of curves in polar coordinates. Reduction formulae, derivations and illustrations of reduction formulae of the type , , , , ,

[1]: Chapter 9 (Section 9.4)

[2]: Chapter 11(Section 11.1), Chapter 8 (Sections 8.2-8.3, pages 532-538)

Volumes by slicing; disks and washers methods, Volumes by cylindrical shells. Arc length, arc length of parametric curves, Area of surface of revolution

[2]: Chapter 6 (Sections 6.2-6.5)

Techniques of sketching conics, reflection properties of conics, Rotation of axes and second degree equations, classification into conics using the discriminant

[2]: Chapter 11 (Section 11.4, 11.5) (Statements of Theorems 11.5.1 and 11.5.2)

Introduction to vector functions and their graphs, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions. Modeling ballistics and planetary motion, Kepler's second law. Curvature, tangential and normal components of acceleration.

[1]: Chapter 10 (Sections 10.1-10.4)

[2]: Chapter 13 (Section 13.5)

Practical / Lab work to be performed on a computer:

Modeling of the following problems using Matlab / Mathematica / Maple etc.

1. Plotting of graphs of function of type (greatest integer function), $y = \lfloor x \rfloor$, $y = \lceil x \rceil$, (even and odd positive integer), $y = \lfloor x \rfloor + 1$, $y = \lceil x \rceil - 1$, (a positive integer) $y = \lfloor x \rfloor + a$, $y = \lceil x \rceil - a$, Discuss the effect of a and x on the graph.
2. Plotting the graphs of polynomial of degree 4 and 5, the derivative graph, the second derivative graph and comparing them.
3. Sketching parametric curves.
4. Tracing of conics in Cartesian coordinates.
5. Obtaining surface of revolution of curves.
6. Sketching ellipsoid, hyperboloid of one and two sheets, elliptic cone, elliptic paraboloid, hyperbolic paraboloid using Cartesian co-ordinates.
7. To find numbers between two real numbers and plotting of finite and infinite subset of \mathbb{R} .
8. Matrix operations (addition, multiplication, inverse, transpose, determinant, rank, eigenvectors, eigenvalues, Characteristic equation and verification of Cayley Hamilton equation, system of linear equations)
9. Graph of Hyperbolic functions.
10. Computation of limit, differentiation and integration of vector functions.
11. Complex numbers and their representations, operations like addition, multiplication, division, modulus. Graphical representation of polar form.

REFERENCES:

1. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
2. H. Anton, I. Bivens and S. Davis, Calculus (7th Edition), John Wiley and sons (Asia), Pt Ltd., Singapore, 2002.

C2- Algebra

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Polar representation of complex numbers, n th roots of unity, De Moivre's theorem for rational indices and its applications.

[1]: Chapter 2

Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

[2]: Chapter 2 (Section 2.4), Chapter 3, Chapter 4 (Sections 4.1 up to 4.1.6, 4.2 up to 4.2.11, 4.4 (till 4.4.8), 4.3.7 to 4.3.9), Chapter 5 (5.1.1, 5.1.4).

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation $Ax = b$, solution sets of linear systems, applications of linear systems, linear independence. Introduction to linear transformations, matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices. Subspaces of \mathbb{R}^n , dimension of subspaces of \mathbb{R}^n and rank of a matrix, Eigen values, Eigen Vectors and Characteristic Equation of a matrix.

[3]: Chapter 1 (Sections 1.1-1.9), Chapter 2 (Sections 2.1-2.3, 2.8-2.9), Chapter 5 (Sections 5.1, 5.2).

REFERENCES:

1. Titu Andreescu and Dorin Andrica, Complex Numbers from A to ... Z, Birkhauser, 2006.
2. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory(3rd Edition), Pearson Education (Singapore) Pvt. Ltd., Indian Reprint, 2005.
3. David C. Lay, Linear Algebra and its Applications (3rd Edition), Pearson Education Asia, Indian Reprint, 2007.

C3- Real Analysis

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Algebraic and Order Properties of R , d -neighborhood of a point in R , Idea of countable sets, uncountable sets and uncountability of R .

[1]: Chapter 1 (Section 1.3), Chapter 2 (Sections 2.1, 2.2.7, 2.2.8)

Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of R , The Archimedean Property, Density of Rational (and Irrational) numbers in R , Intervals.

[1]: Chapter 2 (Sections 2.3, 2.4, 2.5.)

Limit points of a set, Isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.

[1]: Chapter 4 (Section 4.1)

Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano Weierstrass Theorem for Sequences. Cauchy sequence, Cauchy's Convergence Criterion.

[1]: Chapter 3 (Section 3.1-3.5)

Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's n th root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence.

[2]: Chapter 6 (Section 6.2)

REFERENCES:

1. R.G. Bartle and D. R. Sherbert, *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.

2. Gerald G. Bilodeau , Paul R. Thie, G.E. Keough, *An Introduction to Analysis*, Jones & Bartlett, Second Edition, 2010.
3. Brian S. Thomson, Andrew. M. Bruckner, and Judith B. Bruckner, *Elementary Real Analysis*, Prentice Hall, 2001.

C4- Differential Equations (including practicals)

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practical (each in group of 15-20)

Differential equations and mathematical models, order and degree of a differential equation, exact differential equations and integrating factors of first order differential equations, reducible second order differential equations, application of first order differential equations to acceleration-velocity model, growth and decay model.

[2]: Chapter 1 (Sections 1.1, 1.4, 1.6), Chapter 2 (Section 2.3)

[3]: Chapter 2.

Introduction to compartmental models, lake pollution model (with case study of Lake Burley Griffin), drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, case study of alcohol in the bloodstream), exponential growth of population, limited growth of population, limited growth with harvesting.

[1]: Chapter 2 (Sections 2.1, 2.5-2.8), Chapter 3 (Sections 3.1-3.3)

General solution of homogeneous equation of second order, principle of superposition for a homogeneous equation, Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters, applications of second order differential equations to mechanical vibrations.

[2]: Chapter 3 (Sections 3.1-3.5).

Equilibrium points, interpretation of the phase plane, predator-prey model and its analysis, competing species and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.

[1]: Chapter 5 (Sections 5.1, 5.3-5.4, 5.6-5.7), Chapter 6.

Practical / Lab work to be performed on a computer:

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

1. Plotting of second order solution family of differential equation.
2. Plotting of third order solution family of differential equation.
3. Growth model (exponential case only).
4. Decay model (exponential case only).
5.
 - (a) Lake pollution model (with constant/seasonal flow and pollution concentration).
 - (b) Case of single cold pill and a course of cold pills.
 - (c) Limited growth of population (with and without harvesting).
6.
 - (a) Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
 - (b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
 - (c) Battle model (basic battle model, jungle warfare, long range weapons).
7. Plotting of recursive sequences.
8. Find a value of ϵ that will make the following inequality holds for all n :
 - (i) $|x_n - L| < \epsilon$, (ii) $|x_n - L| < \epsilon$,
 - (ii) $|x_n - L| < \epsilon$, (iv) $|x_n - L| < \epsilon$ etc.
9. Study the convergence of sequences through plotting.
10. Verify Bolzano Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.
11. Study the convergence/divergence of infinite series by plotting their sequences of partial sum.

12. Cauchy's root test by plotting n th roots.
13. Ratio test by plotting the ratio of n th and $n+1$ th term.
14. For the following sequences $\langle a_n \rangle$, given $\lim_{n \rightarrow \infty} a_n = L$. Find $\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$ such that

(a)

(b)

(c)

(d)

(e)

15. For the following series $\sum_{n=1}^{\infty} a_n$, calculate

$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$, and identify the convergent series (a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(j)

(k)

(l)

REFERENCES:

1. **Belinda Barnes** and **Glenn R. Fulford**, *Mathematical Modeling with Case Studies, A Differential Equation Approach Using Maple*, Taylor and Francis, London and New York, 2002.
2. **C. H. Edwards** and **D. E. Penny**, *Differential Equations and Boundary Value Problems: Computing and Modeling*, Pearson Education, India, 2005.
3. **S. L. Ross**, *Differential Equations*, John Wiley and Sons, India, 2004.

C5 Theory of Real Functions

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Limits of functions (epsilon-delta approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits & limits at infinity.

[1] Chapter 4, Section 4.1, Section 4.2, Section 4.3 (4.3.1 - 4.3.16)

Continuous functions, sequential criterion for continuity & discontinuity. Algebra of continuous functions.

[1] Chapter 5, Section 5.1, 5.2

Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem.

[2] Art. 18.1, 18.2, 18.3, 18.5, 18.6

Uniform continuity, non-uniform continuity criteria, uniform continuity theorem.

[1] Chapter 5, Section 5.4 (5.4.1 to 5.4.3)

Differentiability of a function at a point & in an interval, Carathéodory's theorem, algebra of differentiable functions.

[1] Chapter 6, Section 6.1 (6.1.1 to 6.1.7)

Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem. Applications of mean value theorem to inequalities & approximation of polynomials Taylor's theorem to inequalities.

[1] Chapter 6, Section 6.2 (6.2.1 to 6.2.7, 6.2.11, 6.2.12)

Cauchy's mean value theorem. Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, application of Taylor's theorem to convex functions, relative extrema. Taylor's series & Maclaurin's series expansions of exponential & trigonometric functions,

[1] Chapter 6, Section 6.3 (6.3.2) Section 6.4 (6.4.1 to 6.4.6)

REFERENCES:

1. **R. G. Bartle & D.R. Sherbert**, Introduction to Real Analysis, John Wiley & Sons (2003)
2. **K. A. Ross**, Elementary Analysis: The Theory of Calculus, Springer (2004).

Suggestive Readings

1. **A. Mattuck**, Introduction to Analysis, Prentice Hall (1999).
2. **S. R. Ghorpade & B. V. Limaye**, A Course in Calculus and Real Analysis – Springer (2006).

C6 Group Theory –I

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups. Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups. Properties of cyclic groups, classification of subgroups of cyclic groups.

[1]: Chapters 1, Chapter 2, Chapter 3 (including Exercise 20 on page 66 and Exercise 2 on page 86), Chapter 4.

Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem. External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.

[1]: Chapter 5 (till end of Theorem 5.7), Chapter 7 (till end of Theorem 7.2, including Exercises 6 and 7 on page 168), Chapter 8 (till the end of Example 2), Chapter 9 (till end of Example 10, Theorem 9.3 and 9.5).

Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, First, Second and Third isomorphism theorems.

[1]: Chapter 6 (till end of Theorem 6.2), Chapter 10.

REFERENCES:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.(IX Edition 2010)

SUGGESTED READING:

1. Joseph J. Rotman, *An Introduction to the Theory of Groups* (4th Edition), Springer Verlag, 1995.

C7 Multivariate Calculus (including practicals)

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practical (each in group of 15-20)

Functions of several variables, limit and continuity of functions of two variables. Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability. Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes

[1]: Chapter 11 (Sections 11.1(Pages 541-543), 11.2-11.6)

Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Definition of vector field, divergence and curl

[1]: Chapter 11(Sections 11.7 (Pages 598-605), 11.8(Pages 610-614))

Chapter 13 (Pages 684-689)

Double integration over rectangular region, double integration over nonrectangular region. Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals.

[1]: Chapter 12 (Sections 12.1, 12.2, 12.3, 12.4 (Pages 652-660), 12.5, 12.6)

Line integrals, Applications of line integrals: Mass and Work. Fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stokes' theorem, The Divergence theorem.

[1]: Chapter 13 (Section 13.2, 13.3, 13.4(Page 712–716), 13.5(Page 723–726, 729-730), 13.6 (Page 733–737), 13.7 (Page 742–745))

REFERENCES:

1. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

SUGGESTED READING:

2. E. Marsden, A. J. Tromba and A. Weinstein, *Basic multivariable calculus*, Springer (SIE), Indian reprint, 2005.

Practical / Lab work to be performed on a computer:

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

1. Draw the following surfaces and find level curves at the given heights:

- (i) $z = 2x^2 + 3y^2$, $z = 1$
- (ii) $z = 4x^2 - y^2$, $z = 1$, (iii) $z = 2x^2 - 3y^2$, $z = 1$
- (iv) $z = 2x^2 + 3y^2 + 4z^2$, $z = 1$
- (v) $z = 2x^2 + 3y^2 + 4z^2$, $z = 1$
- (vi) $z = 2x^2 + 3y^2 + 4z^2$, $z = 1$

2. Draw the following surfaces and discuss whether limit exists or not as approaches to the given points. Find the limit, if it exists:

- (i) $z = 2x^2 + 3y^2$, $(1, 1, 5)$
- (ii) $z = 4x^2 - y^2$, $(1, 1, 3)$
- (iii) $z = 2x^2 - 3y^2$, $(1, 1, 1)$
- (iv) $z = 2x^2 + 3y^2 + 4z^2$, $(1, 1, 1)$
- (v) $z = 2x^2 + 3y^2 + 4z^2$, $(1, 1, 1)$

(vi) $z = 2x^2 + 3y^2 + 4z^2$, $(1, 1, 1)$

(vii) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(viii)

3. Draw the tangent plane to the following surfaces at the given point:

(i) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$ (ii) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(iii) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(iv) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(v) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

4. Use an incremental approximation to estimate the following functions at the given point and compare it with calculated value:

(i) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$ (ii) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(iii) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

(iv) $z = x^2 + y^2 + z^2$, $(1, 1, 1)$

5. Find critical points and identify relative maxima, relative minima or saddle points to the following surfaces, if it exist:

(i) $z = x^2 + y^2 + z^2$, (ii) $z = x^2 + y^2 + z^2$, (iii) $z = x^2 + y^2 + z^2$, (iv) $z = x^2 + y^2 + z^2$

6. Draw the following regions **D** and check whether these regions are of **Type I** or **Type II**:

(i) $\lim_{x \rightarrow a} f(x) = L$,

(ii) $\lim_{x \rightarrow a} f(x) = L$,

7. Illustrations of the following :

1. Let $f(x)$ be any function and a be any number. For given $\epsilon > 0$ and $\delta > 0$, find a δ such that for all x satisfying $0 < |x - a| < \delta$, the inequality $|f(x) - L| < \epsilon$ holds. For examples:

(i)

(ii)

(iii)

(iv)

8. Discuss the limit of the following functions when x tends to

0:

9. Discuss the limit of the following functions when x tends to infinity :

10. Discuss the continuity of the functions at $x = a$ in practical 2.

11. Illustrate the geometric meaning of Rolle's theorem of the following functions on the given interval :

12. Illustrate the geometric meaning of Lagrange's mean value theorem of the following functions on the given interval:

13. For the following functions and given ϵ , if exists, find δ such that $|f(x) - f(y)| < \epsilon$ and discuss uniform continuity of the functions:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

14. Verification of Maximum –Minimum theorem, boundedness theorem & intermediate value theorem for various functions and the failure of the conclusion in case of any of the hypothesis is weakened.

15. Locating points of relative & absolute extremum for different functions

16. Relation of monotonicity & derivatives along with verification of first derivative test.

17. Taylor's series - visualization by creating graphs:

- a. Verification of simple inequalities
- b. Taylor's Polynomials – approximated up to certain degrees
- c. Convergence of Taylor's series
- d. Non-existence of Taylor series for certain functions
- e. Convexity of the curves

C8 Partial Differential Equations (including practicals)

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practical (each in group of 15-20)

Introduction, classification, construction and geometrical interpretation of first order partial differential equations (PDE), method of characteristic and general solution of first order PDE, canonical form of first order PDE, method of separation of variables for first order PDE.

[1]: Chapter 2.

Mathematical modeling of vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws and Burger's equations, classification of second order PDE, reduction to canonical forms, equations with constant coefficients, general solution.

[1]: Chapter 3 (Sections 3.1-3.3, 3.5-3.7), Chapter 4.

Cauchy problem for second order PDE, homogeneous wave equation, initial boundary value problems, non-homogeneous boundary conditions, finite strings with fixed ends, non-homogeneous wave equation, Riemann problem, Goursat problem, spherical and cylindrical wave equation.

[1]: Chapter 5.

Method of separation of variables for second order PDE, vibrating string problem, existence and uniqueness of solution of vibrating string problem, heat conduction

problem, existence and uniqueness of solution of heat conduction problem, Laplace and beam equation, non-homogeneous problem.

[1]: Chapter 7.

Practical / Lab work to be performed on a computer:

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

1. Solution of Cauchy problem for first order PDE.
2. Plotting the characteristics for the first order PDE.
3. Plot the integral surfaces of a given first order PDE with initial data.

4. Solution of wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for any 2 of the following associated conditions:

(a) $u(0, t) = f(t), u(l, t) = g(t)$

(b) $u(0, t) = f(t), u_x(0, t) = g(t)$

(c) $u_x(0, t) = f(t), u_x(l, t) = g(t)$

(d) $u(0, t) = f(t), u_x(l, t) = g(t)$

5. Solution of one-Dimensional heat equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, for a homogeneous rod of length l .

That is - solve the IBVP:

6. Solving systems of ordinary differential equations.

7. Approximating solution to Initial Value Problems using any of the following approximate methods:

(a) The Euler Method

(b) The Modified Euler Method.

(c) The Runge-Kutta Method.

Comparison between exact and approximate results for any representative differential equation.

8. Draw the following sequence of functions on given the interval and discuss the pointwise convergence:

(i) $f_1(x) = x$, (ii) $f_2(x) = x^2$,

(iii) $f_3(x) = x^3$, (iv) $f_4(x) = x^4$,

(v) $f_5(x) = x^5$, (vi) $f_6(x) = x^6$,

(vii) $f_7(x) = x^7$,

(viii) $f_8(x) = x^8$,

9. Discuss the uniform convergence of sequence of functions above.

REFERENCE:

1. **Tyn Myint-U** and **Lokenath Debnath**, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.

SUGGESTED READING:

1. **Ioannis P Stavroulakis** and **Stepan A Tersian**, *Partial Differential Equations: An Introduction with Mathematica and MAPLE*, World Scientific, Second Edition, 2004.

C9 Riemann Integration & Series of Functions

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorems of Calculus.

[1] Chapter 6 (Art. 32.1 to 32.9, 33.1, 33.2, 33.3, 33.4 to 33.8, 33.9, 34.1, 34.3)

Improper integrals; Convergence of Beta and Gamma functions.

[3] Chapter 7 (Art. 7.8)

Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions.

[2] Chapter 8, Section 8.1, Section 8.2 (8.2.1 – 8.2.2), Theorem 8.2.3, Theorem 8.2.4 and Theorem 8.2.5

Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test

[2] Chapter 9, Section 9.4 (9.4.1 to 9.4.6)

Limit superior and Limit inferior. Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem.

[1] Chapter 4, Art. 26 (26.1 to 26.6), Theorem 27.5

REFERENCES:

1. K.A. Ross, Elementary Analysis: The Theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
2. R.G. Bartle D.R. Sherbert, Introduction to Real Analysis (3rd edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
3. Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition), 2011.

C 10 Ring Theory & Linear Algebra-I

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lecture, 1 Tutorial (per week per student)

Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients.

[2]: Chapter 12, Chapter 13, Chapter 14, Chapter 15.

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.

[1]: Chapter 1 (Sections 1.2-1.6, Exercise 29, 33, 34, 35), Chapter 2 (Sections 2.1-2.5).

REFERENCES:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
2. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.

SUGGESTED READING:

1. S Lang, *Introduction to Linear Algebra* (2nd edition), Springer, 2005
2. Gilbert Strang, *Linear Algebra and its Applications*, Thomson, 2007
3. S. Kumaresan, *Linear Algebra- A Geometric Approach*, Prentice Hall of India, 1999.
4. Kenneth Hoffman, Ray Alden Kunze, *Linear Algebra* 2nd Ed., Prentice-Hall Of India Pvt. Limited, 1971

C 11 Metric Spaces

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Metric spaces: definition and examples. Sequences in metric spaces, Cauchy sequences. Complete Metric Spaces.

[1] Chapter1, Section 1.2 (1.2.1 to 1.2.6). Section 1.3, Section 1.4 (1.4.1 to 1.4.4), Section 1.4 (1.4.5 to 1.4.14 (ii)).

Open and closed balls, neighbourhood, open set, interior of a set, Limit point of a set, closed set, diameter of a set, Cantor's Theorem, Subspaces, dense sets, separable spaces.

[1] Chapter2, Section 2.1 (2.1.1 to 2.1.16), Section 2.1 (2.1.17 to 2.1.44), Section 2.2, Section 2.3 (2.3.12 to 2.3.16)

Continuous mappings, sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mappings, Banach Fixed point Theorem.

[1] Chapter3, Section 3.1, Section3.4 (3.4.1 to 3.4.8), Section 3.5 (3.5.1 to 3.5.7(iv)), Section 3.7 (3.7.1 to 3.7.5)

Connectedness, connected subsets of \mathbf{R} , connectedness and continuous mappings.

[1] Chapter4, Section 4.1 (4.1.1 to 4.1.12)

Compactness, compactness and boundedness, continuous functions on compact spaces.

[1] Chapter5, Section 5.1 (5.1.1 to 5.1.6), Section 5.3 (5.3.1 to 5.3.11)

REFERENCES:

[1] Satish Shirali & Harikishan L. Vasudeva, Metric Spaces, Springer Verlag London (2006) (First Indian Reprint 2009)

SUGGESTED READINGS:

[1] S. Kumaresan, Topology of Metric Spaces, Narosa Publishing House, Second Edition 2011.

[2] G. F. Simmons, Introduction to Topology and Modern Analysis, Mcgraw-Hill, Edition 2004.

C 12 Group Theory-II

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties.

[1]: Chapter 6, Chapter 9 (Theorem 9.4), Exercises 1-4 on page 168, Exercises 52, 58 on page Pg 188.

Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups.

[1]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

Group actions, stabilizers and kernels, permutation representation associated with a given group action, Applications of group actions: Generalized Cayley's theorem, Index theorem. Groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n , p -groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests.

[2]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1-4.3, 4.5-4.6).

[1]: Chapter 25.

REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.

2. David S. Dummit and Richard M. Foote, Abstract Algebra (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2004

C13 Complex Analysis (including practicals)

Total marks: 150

Theory: 75

Internal Assessment: 25

Practical: 50

5 Lectures, Practical 4 (in group of 15-20)

Limits, Limits involving the point at infinity, continuity.

Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

[1]: Chapter 1 (Section 11), Chapter 2 (Section 12, 13) Chapter 2 (Sections 15, 16, 17, 18, 19, 20, 21, 22)

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions.

[1]: Chapter 2 (Sections 24, 25), Chapter 3 (Sections 29, 30, 34), Chapter 4 (Section 37, 38)

Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals.

[1]: Chapter 4 (Section 39, 40, 41, 43)

Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula. An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.

[1]: Chapter 4 (Sections 44, 45, 46, 50), Chapter 4 (Sections 51, 52, 53)

Convergence of sequences and series, Taylor series and its examples. Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series.

[1]: Chapter 5 (Sections 55, 56, 57, 58, 59, 60, 62, 63, 66)

Isolated singular points, residues, Cauchy's residue theorem, residue at infinity. Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.

[1]: Chapter 6 (Sections 68, 69, 70, 71, 72, 73, 74), Chapter 7 (Section 85).

REFERENCES:

1. James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications* (Eighth Edition), McGraw – Hill International Edition, 2009.

SUGGESTED READING:

2. Joseph Bak and Donald J. Newman, *Complex analysis* (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

**LAB WORK TO BE PERFORMED ON A COMPUTER
(MODELING OF THE FOLLOWING PROBLEMS USING MATLAB/ MATHEMATICA/
MAPLE ETC.)**

1. Declaring a complex number and graphical representation.

e.g. $Z_1 = 3 + 4i$, $Z_2 = 4 - 7i$

2. Program to discuss the algebra of complex numbers.

e.g., if $Z_1 = 3 + 4i$, $Z_2 = 4 - 7i$, then find $Z_1 + Z_2$, $Z_1 - Z_2$, $Z_1 * Z_2$, and Z_1 / Z_2

3. To find conjugate, modulus and phase angle of an array of complex numbers.

e.g., $Z = [2+ 3i \quad 4-2i \quad 6+11i \quad 2-5i]$

4. To compute the integral over a straight line path between the two specified end points.

e. g., $\int_C f(z) dz$, where C is the straight line path from $-1+ i$ to $2 - i$.

5. To perform contour integration.

e.g., (i) $\int_C z^n dz$, where C is the Contour given by $x = y^2 + 1$; $y = 0$ to $y = 1$.

(ii) $\int_C e^{iz} dz$, where C is the contour given by $|z| = 1$, which can be parameterized by $x = \cos (t)$, $y = \sin (t)$ for $t = 0$ to $t = 2\pi$.

6. To plot the complex functions and analyze the graph .

e.g., (i) $f(z) = Z$

(ii) $f(z) = Z^3$

4. $f(z) = (Z^4 - 1)^{1/4}$

5. , etc.

7. To perform the Taylor series expansion of a given function $f(z)$ around a given point z .

The number of terms that should be used in the Taylor series expansion is given for each function. Hence plot the magnitude of the function and magnitude of its Taylor series expansion.

e.g., (i) $f(z) = \exp(z)$ around $z = 0$, $n = 40$.

(ii) $f(z) = \exp(z^2)$ around $z = 0$, $n = 160$.

8. To determine how many terms should be used in the Taylor series expansion of a given function $f(z)$ around $z = 0$ for a specific value of z to get a percentage error of less than 5 %.

e.g., For $f(z) = \exp(z)$ around $z = 0$, execute and determine the number of necessary terms to get a percentage error of less than 5 % for the following values of z :

(i) $z = 30 + 30i$

(ii)

9. To perform Laurent series expansion of a given function $f(z)$ around a given point z .

e.g., (i) $f(z) = (\sin z - 1)/z^4$ around $z = 0$

(ii) $f(z) = \cot(z)/z^4$ around $z = 0$.

10. To compute the poles and corresponding residues of complex functions.

e.g.,

11. To perform Conformal Mapping and Bilinear Transformations.

C 14 Ring Theory and Linear Algebra – II

Total Marks : 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in $\mathbb{Z}[x]$.

Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

[1]: Chapter 16, Chapter 17, Chapter 18.

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators, Eigenspaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator.

[2]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4), Chapter 7(Section 7.3).

Inner product spaces and norms, Gram-Schmidt orthogonalization process, orthogonal complements, Bessel's inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.

[2]: Chapter 6 (Sections 6.1-6.4, 6.6).

REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

SUGGESTED READING:

(Linear Algebra)

1. S Lang, Introduction to Linear Algebra (2nd edition), Springer, 2005
2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007

3. S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.
4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra 2nd Ed., Prentice-Hall Of India Pvt. Limited, 1971

(Ring theory and group theory)

1. John B.Fraleigh, A first course in Abstract Algebra, 7th Edition, Pearson Education India, 2003.
2. Herstein, Topics in Algebra (2nd edition), John Wiley & Sons, 2006
3. Michael Artin, Algebra (2nd edition), Pearson Prentice Hall, 2011
4. Robinson, Derek John Scott., An introduction to abstract algebra, Hindustan book agency, 2010.

DSE-1 (including practicals): Any one of the following (at least two shall be offered by the college):

DSE-1(i) Numerical Methods

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practicals (each in group of 15-20)

Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.

[1]: Chapter 1 (Sections 1.1-1.2), Chapter 2 (Sections 2.1-2.5), Chapter 3 (Section 3.5, 3.8).

Lagrange and Newton interpolation: linear and higher order, finite difference operators.

[1]: Chapter 5 (Sections 5.1, 5.3)

[2]: Chapter 4 (Section 4.3).

Numerical differentiation: forward difference, backward difference and central difference. Integration: trapezoidal rule, Simpson's rule, Euler's method.

[1]: Chapter 6 (Sections 6.2, 6.4), Chapter 7 (Section 7.2)

Note: Emphasis is to be laid on the algorithms of the above numerical methods.

Practical / Lab work to be performed on a computer:

Use of computer aided software (CAS), for example *Matlab / Mathematica / Maple / Maxima* etc., for developing the following Numerical programs:

(i) Calculate the sum $1/1 + 1/2 + 1/3 + 1/4 + \dots + 1/N$.

(ii) To find the absolute value of an integer.

- (iii) Enter 100 integers into an array and sort them in an ascending order.
- (iv) Any two of the following
 - (a) Bisection Method
 - (b) Newton Raphson Method
 - (c) Secant Method
 - (d) Regulai Falsi Method
- (v) LU decomposition Method
- (vi) Gauss-Jacobi Method
- (vii) SOR Method or Gauss-Siedel Method
- (viii) Lagrange Interpolation or Newton Interpolation
- (ix) Simpson's rule.

Note: For any of the CAS *Matlab / Mathematica / Maple / Maxima* etc., Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

REFERENCES:

1. **B. Bradie**, *A Friendly Introduction to Numerical Analysis*, Pearson Education, India, 2007.
2. **M. K. Jain, S. R. K. Iyengar and R. K. Jain**, *Numerical Methods for Scientific and Engineering Computation*, New age International Publisher, India, 5th edition, 2007.

SUGGESTED READING:

1. **C. F. Gerald and P. O. Wheatley**, *App;ied Numerical Analysis*, Pearson Education, India, 7th edition, 2008

DSE-1(ii) Mathematical Modeling & Graph Theory

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practicals (each in group of 15-20)

Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

[2]: Chapter 7 (Sections 7.1-7.3), Chapter 8 (Sections 8.2-8.3).

Monte Carlo Simulation Modeling: simulating deterministic behavior (area under a curve, volume under a surface), Generating Random Numbers: middle square method, linear congruence, Queuing Models: harbor system, morning rush hour, Overview of optimization modeling, Linear Programming Model: geometric solution algebraic solution, simplex method, sensitivity analysis

[3]: Chapter 5 (Sections 5.1-5.2, 5.5), Chapter 7.

Graphs, diagraphs, networks and subgraphs, vertex degree, paths and cycles, regular and bipartite graphs, four cube problem, social networks, exploring and traveling, Eulerian and Hamiltonian graphs, applications to dominoes, diagram tracing puzzles, Knight's tour problem, gray codes.

[1]: Chapter 1 (Section 1.1), Chapter 2, Chapter 3.

Note: Chapter 1 (Section 1.1), Chapter 2 (Sections 2.1-2.4), Chapter 3 (Sections 3.1-3.3) are to be reviewed only. This is in order to understand the models on Graph Theory.

Practical / Lab work to be performed on a computer:

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

(i) Plotting of Legendre polynomial for $n = 1$ to 5 in the interval $[0,1]$. Verifying graphically that all the roots of $P_n(x)$ lie in the interval $[0,1]$.

- (ii) Automatic computation of coefficients in the series solution near ordinary points
- (iii) Plotting of the Bessel's function of first kind of order 0 to 3.
- (iv) Automating the Frobenius Series Method
- (v) Random number generation and then use it for one of the following
 - (a) Simulate area under a curve
 - (b) Simulate volume under a surface
- (vi) Programming of either one of the queuing model
 - (a) Single server queue (e.g. Harbor system)
 - (b) Multiple server queue (e.g. Rush hour)
- (vii) Programming of the Simplex method for 2/3 variables

REFERENCES:

1. **Joan M. Aldous** and **Robin J. Wilson**, *Graphs and Applications: An Introductory Approach*, Springer, Indian reprint, 2007.
2. **Tyn Myint-U** and **Lokenath Debnath**, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.
3. **Frank R. Giordano**, **Maurice D. Weir** and **William P. Fox**, *A First Course in Mathematical Modeling*, Thomson Learning, London and New York, 2003.

DSE-1(iii) C++ PROGRAMMING

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 4 Practicals (each in group of 15-20)

Introduction to structured programming: data types- simple data types, floating data types, character data types, string data types, arithmetic operators and operators precedence, variables and constant declarations, expressions, input using the extraction operator >> and cin, output using the insertion operator << and cout, preprocessor directives, increment(++) and decrement(--) operations, creating a C++ program, input/ output, relational operators, logical operators and logical expressions, if and if-else statement, switch and break statements.

[1]Chapter 2(pages 37-95), Chapter3(pages 96 -129), Chapter 4(pages 134-178)

“for”, “while” and “do-while” loops and continue statement, nested control statement, value returning functions, value versus reference parameters, local and global variables, one dimensional array, two dimensional array, pointer data and pointer variables,.

[1] Chapter 5 (pages 181 - 236), Chapter 6, Chapter 7(pages 287- 304)Chapter 9 (pages 357 - 390), Chapter 14 (pages 594 - 600).

Reference:

[1]D. S. Malik: C++ Programming Language, Edition-2009, Course Technology, Cengage Learning, India Edition

Suggested Readings:

[2]E. Balaguruswami: Object oriented programming with C++, fifth edition, Tata McGraw Hill Education Pvt. Ltd.

[3]Marshall Cline, Greg Lomow, Mike Girou: C++ FAQs, Second Edition, Pearson Education.

Note: Practical programs of the following (and similar) type are suggestive.

1. Calculate the Sum of the series $1/1 + 1/2 + 1/3 + \dots + 1/N$ for any positive integer N.
2. Write a user defined function to find the absolute value of an integer and use it to evaluate the function $(-1)^n/|n|$, for $n = -2, -1, 0, 1, 2$.
3. Calculate the factorial of any natural number.
4. Read floating numbers and compute two averages: the average of negative numbers and the average of positive numbers.
5. Write a program that prompts the user to input a positive integer. It should then output a message indicating whether the number is a prime number.
6. Write a program that prompts the user to input the value of a, b and c involved in the equation $ax^2 + bx + c = 0$ and outputs the type of the roots of the equation. Also the program should outputs all the roots of the equation.
7. write a program that generates random integer between 0 and 99. Given that first two Fibonacci numbers are 0 and 1, generate all Fibonacci numbers less than or equal to generated number.
8. Write a program that does the following:
 - a. Prompts the user to input five decimal numbers.
 - b. Prints the five decimal numbers.
 - c. Converts each decimal number to the nearest integer.
 - d. Adds these five integers.
 - e. Prints the sum and average of them.
9. Write a program that uses **while** loops to perform the following steps:
 - a. Prompt the user to input two integers :firstNum and secondNum (firstNum should be less than secondNum).
 - b. Output all odd and even numbers between firstNum and secondNum.
 - c. Output the sum of all even numbers between firstNum and secondNum.
 - d. Output the sum of the square of the odd numbers firstNum and secondNum.
 - e. Output all uppercase letters corresponding to the numbers between firstNum and secondNum, if any.
10. Write a program that prompts the user to input five decimal numbers. The program should then add the five decimal numbers, convert the sum to the nearest integer, and print the result.
11. Write a program that prompts the user to enter the lengths of three sides of a triangle and then outputs a message indicating whether the triangle is a right triangle or a scalene triangle.

12. Write a value returning function ***smaller*** to determine the smallest number from a set of numbers. Use this function to determine the smallest number from a set of 10 numbers.
13. Write a function that takes as a parameter an integer (as a **long** value) and returns the number of odd, even, and zero digits. Also write a program to test your function.
14. Enter 100 integers into an array and sort them in an ascending/ descending order and print the largest/ smallest integers.
15. Enter 10 integers into an array and then search for a particular integer in the array.
16. Multiplication/ Addition of two matrices using two dimensional arrays.
17. Using arrays, read the vectors of the following type: $A = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)$, $B = (0\ 2\ 3\ 4\ 0\ 1\ 5\ 6)$ and compute the product and addition of these vectors.
18. Read from a text file and write to a text file.
19. Write a program to create the following grid using for loops:

```
1 2 3 4 5
2 3 4 5 6
3 4 5 6 7
4 5 6 7 8
5 6 7 8 9
```
20. Write a function, *reverseDigit*, that takes an integer as a parameter and returns the number with its digits reversed. For example, the value of function *reverseDigit*(12345) is 54321 and the value of *reverseDigit*(-532) is -235.

DSE-2: Any one of the following (at least two shall be offered by the college):

DSE-2(i) Mathematical Finance

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, puttable and callable bonds.

[1]: Chapter 1, Chapter 2, Chapter 3, Chapter 4.

Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index. Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.

[1]: Chapter 6, Chapter 7, Chapter 8 (Sections 8.5--8.8).

[3]: Chapter 1 (for a quick review/description of expectation etc.)

Forwards and futures, marking to market, value of a forward/futures contract, replicating portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps. Lognormal distribution, Lognormal model / Geometric Brownian Motion for stock prices, Binomial Tree model for stock prices, parameter estimation, comparison of the models. Options, Types of options: put / call, European / American, pay off of an option, factors affecting option prices, put call parity.

[1]: Chapter 10 (except 10.11, 10.12), Chapter 11 (except 11.2 and 11.8)

[2]: Chapter 3, Chapter 5, Chapter 6, Chapter 7 (except 7.10 and 7.11), Chapter 8, Chapter 9

[3]: Chapter 3

REFERENCES:

1. **David G. Luenberger**, *Investment Science*, Oxford University Press, Delhi, 1998.
2. **John C. Hull**, *Options, Futures and Other Derivatives* (6th Edition), Prentice-Hall India, Indian reprint, 2006.
3. **Sheldon Ross**, *An Elementary Introduction to Mathematical Finance* (2nd Edition), Cambridge University Press, USA, 2003.

DSE-2(ii) Discrete Mathematics

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms.

[1]: Chapter 1 (till the end of 1.18), Chapter 2 (Sections 2.1-2.13), Chapter 5 (Sections 5.1-5.11).

[3]: Chapter 1 (Section 1).

Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

[1]: Chapter 6.

[3]: Chapter 1 (Sections 3-4, 6), Chapter 2 (Sections 7-8).

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bipartite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

[2]: Chapter 9, Chapter 10.

REFERENCES:

1. **B A. Davey** and **H. A. Priestley**, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1990.
2. **Edgar G. Goodaire** and **Michael M. Parmenter**, *Discrete Mathematics with Graph Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint 2003.
3. **Rudolf Lidl** and **Günter Pilz**, *Applied Abstract Algebra* (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

DSE-2(iii) CRYPTOGRAPHY AND NETWORK SECURITY_

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques- Substitution and transposition ciphers, caesar cipher, Playfair cipher. Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard (DES).

[1] 2.1-2.3, 3.1, 3.2, 3.3.

Polynomial and modular arithmetic, Introduction to finite field of the form $GF(p)$ and $GF(2^n)$, Fermat theorem and Euler's theorem(statement only), Chinese Remainder theorem, Discrete logarithm.

[1] 4.2, 4.3, 4.5, 4.6, 4.7, 8.2, 8.4, 8.5

Advanced Encryption Standard(AES), Stream ciphers . Introduction to public key cryptography, RSA algorithm and security of RSA, Introduction to elliptic curve cryptography.

[1] 5.2-5.5(tables 5.5, 5.6 excluded),7.4, 9.1, 9.2, 10.3, 10.4

Information/Computer Security: Basic security objectives, security attacks, security services, Network security model,

[1]1.1, 1.3, 1.4, 1.6

Cryptographic Hash functions, Secure Hash algorithm, SHA-3.

[1] 11.1, 11.5, 11.6

Digital signature, Elgamal signature, Digital signature standards, Digital signature algorithm

[1] 13.1, 13.2, 13.4

E-mail security: Pretty Good Privacy (PGP)

[1] 18.1 Page 592-596(Confidentiality excluded)

REFERENCE:

[1] William Stallings, "Cryptography and Network Security", Principles and Practise, Fifth Edition, Pearson Education, 2012.

SUGGESTED READING:

[1] Douglas R. Stinson, "Cryptography theory and practice", CRC Press, Third edition, 2005.

DSE-3: Any one of the following (at least two shall be offered by the college):

DSE-3(i) Probability Theory and Statistics

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Sample space, probability axioms, real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential.

[1]: Chapter 1 (Sections 1.1, 1.3, 1.5-1.9).

[2]: Chapter 5 (Sections 5.1-5.5, 5.7), Chapter 6 (Sections 6.2-6.3, 6.5-6.6).

Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, conditional expectations, independent random variables, bivariate normal distribution, correlation coefficient, joint moment generating

function (jmgf) and calculation of covariance (from jmgf), linear regression for two variables.

[1]: Chapter 2 (Sections 2.1, 2.3-2.5).

[2]: Chapter 4 (Exercise 4.47), Chapter 6 (Section 6.7), Chapter 14 (Sections 14.1, 14.2).

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, Central Limit theorem for independent and identically distributed random variables with finite variance, Markov Chains, Chapman-Kolmogorov equations, classification of states.

[2]: Chapter 4 (Section 4.4).

[3]: Chapter 2 (Section 2.7), Chapter 4 (Sections 4.1-4.3).

REFERENCES:

1. **Robert V. Hogg, Joseph W. McKean and Allen T. Craig**, *Introduction to Mathematical Statistics*, Pearson Education, Asia, 2007.
2. **Irwin Miller and Marylees Miller**, *John E. Freund's Mathematical Statistics with Applications* (7th Edition), Pearson Education, Asia, 2006.
3. **Sheldon Ross**, *Introduction to Probability Models* (9th Edition), Academic Press, Indian Reprint, 2007.

SUGGESTED READING:

1. **Alexander M. Mood, Franklin A. Graybill and Duane C. Boes**, *Introduction to the Theory of Statistics*, (3_{rd} Edition), Tata McGraw- Hill, Reprint 2007

DSE-3(ii) Mechanics

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Moment of a force about a point and an axis, couple and couple moment, Moment of a couple about a line, resultant of a force system, distributed force system, free body diagram, free body involving interior sections, general equations of equilibrium, two point equivalent loading, problems arising from structures, static indeterminacy.

[1]: Chapter 3, Chapter 4, Chapter 5.

Laws of Coulomb friction, application to simple and complex surface contact friction problems, transmission of power through belts, screw jack, wedge, first moment of an area and the centroid, other centers, Theorem of Pappus-Guldinus, second moments and the product of area of a plane area, transfer theorems, relation between second moments and products of area, polar moment of area, principal axes.

[1]: Chapter 6 (Sections 6.1-6.7), Chapter 7

Conservative force field, conservation for mechanical energy, work energy equation, kinetic energy and work kinetic energy expression based on center of mass, moment of momentum equation for a single particle and a system of particles, translation and rotation of rigid bodies, Chasles' theorem, general relationship between time derivatives of a vector for different references, relationship between velocities of a particle for different references, acceleration of particle for different references.

[1]: Chapter 11, Chapter 12 (Sections 12.5-12.6), Chapter 13.

REFERENCES:

1. **I.H. Shames** and **G. Krishna Mohan Rao**, *Engineering Mechanics: Statics and Dynamics* (4th Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.
2. **R.C. Hibbeler** and **Ashok Gupta**, *Engineering Mechanics: Statics and Dynamics* (11th Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi.

DSE-3(iii) Bio-Mathematics

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Population growth, Administration of drugs, Cell division. Modelling Biological Phenomena: Heart beat, Blood Flow, Nerve Impulse transmission, Chemical Reactions, Predator-prey models. Stability and oscillations: Epidemics, the phase plane, Local Stability, Stability, Limit Cycles, Forced oscillations, Computing trajectories. Mathematics of Heart Physiology: The local model, The Threshold effect, The phase plane analysis and the heart beat model, Physiological considerations of the heart beat model, A model of the cardiac pace-maker. Mathematics of Nerve Impulse transmission: Excitability and repetitive firing, travelling waves. Bifurcation and chaos: Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation, Chaos, Stability, The Poincare plane, Computer programs for Iteration Schemes.

References: Relevant sections of chapters 1, 3, 4, 5, 6, 7 and 13 of [4]

Mathematics of imaging of the Brain: Modelling of computerized tomography (CT, Magnetic resonance Imaging (MRI), Positron emission Tomography (PET), Single Photon Emission Computerized Tomography(SPECT), Discrete analogues and Numerical Implementation. Networks in Biological Sciences: Dynamics of Small world networks, scale-free networks, complex networks, cellular automata.

References: Relevant parts of [2] and [3]

Modelling Molecular Evolution: Matrix models of base substitutions for DNA sequences, The Jukes-Cantor Model, the Kimura Models, Phylogenetic distances. Constructing Phylogenetic trees: Unweighted pair-group method with arithmetic means (UPGMA), Neighbour- Joining Method, Maximum Likelihood approaches. Genetics: Mendelian Genetics, Probability distributions in Genetics, Linked genes and Genetic Mapping, Statistical Methods and Prediction techniques.

References: Relevant sections of Chapters 4, 5 and 6 of [1] and chapters 3, 4, 6 and 8 of [5].

Recommended Books:

1. Elizabeth S. Allman and John a. Rhodes, *Mathematical Models in Biology*, Cambridge University Press, 2004.
2. C. Epstein, *The Mathematics of Medical Imaging*, Prentice Hall, 2003 (copyright Pearson Education, 2005).
3. S. Helgason, *The Radon transform*, Second Edition, Birkhauser, 1997.
4. D. S. Jones and B. D. Sleeman, *Differential Equations and Mathematical Biology*, Chapman & Hall, CRC Press, London, UK, 2003.
5. James Keener and James Sneyd, *Mathematical Physiology*, Springer Verlag, 1998, Corrected 2nd printing, 2001.

DSE-4: Any one of the following (at least two shall be offered by the college):

DSE-4(i) Number Theory

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

References:

[1]: Chapter 2 (Section 2.5), Chapters 3 (Section 3.3), Chapter 4 (Sections 4.2 and 4.4), Chapter 5 (Section 5.2 excluding pseudoprimes, Section 5.3).

[2]: Chapter 3 (Section 3.2).

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function.

References:

[1]: Chapter 6 (Sections 6.1-6.3), Chapter 7.

[2]: Chapter 5 (Section 5.2 (Definition 5.5-Theorem 5.40), Section 5.3 (Theorem 5.15-Theorem 5.17, Theorem 5.19)).

Order of an integer modulo n , primitive roots for primes, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadratic congruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation $x^2 + y^2 = z^2$, Fermat's Last Theorem.

Reference:

[1]: Chapters 8 (Sections 8.1-8.3), Chapter 9, Chapter 10 (Section 10.1), Chapter 12.

REFERENCES:

1. **David M. Burton**, *Elementary Number Theory* (6th Edition), Tata McGraw-Hill Edition, Indian reprint, 2007.
2. **Neville Robinns**, *Beginning Number Theory* (2nd Edition), Narosa Publishing House Pvt. Limited, Delhi, 2007.

DSE-4 (ii) Linear Programming and Theory of Games

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Introduction to linear programming problem, Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

[1]: Chapter 3 (Sections 3.2-3.3, 3.5-3.8), Chapter 4 (Sections 4.1-4.4).

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

[1]: Chapter 6 (Sections 6.1- 6.3).

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

[3]: Chapter 5 (Sections 5.1, 5.3-5.4).

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

[2]: Chapter 14.

REFERENCES:

1. **Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali**, *Linear Programming and Network Flows* (2nd edition), John Wiley and Sons, India, 2004.
2. **F. S. Hillier and G. J. Lieberman**, *Introduction to Operations Research- Concepts and Cases* (9th Edition), Tata McGraw Hill, 2010.
3. **Hamdy A. Taha**, *Operations Research, An Introduction* (9th edition), Prentice-Hall, 2010.

SUGGESTED READING:

1. **G. Hadley**, *Linear Programming*, Narosa Publishing House, New Delhi, 2002.

DSE-4(iii) Applications of Algebra

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Balanced incomplete block designs (BIBD): definitions and results, incidence matrix of a BIBD, construction of BIBD from difference sets, construction of BIBD using quadratic residues, difference set families, construction of BIBD from finite fields.

[2]: Chapter 2 (Sections 2.1-2.4,2.6).

Coding Theory: introduction to error correcting codes, linear codes, generator and parity check matrices, minimum distance, Hamming Codes, decoding and cyclic codes.

[2]: Chapter 4 (Sections 4.1-4.3.17).

Symmetry groups and color patterns: review of permutation groups, groups of symmetry and action of a group on a set; colouring and colouring patterns, Polya theorem and pattern inventory, generating functions for non-isomorphic graphs.

[2]: Chapter 5.

Application of linear transformations: Fibonacci numbers, incidence models, and differential equations. Least squares methods: Approximate solutions of system of linear equations, approximate inverse of an $m \times n$ matrix, solving a matrix equation using its normal equation, finding functions that approximate data. Linear algorithms: LDU factorization, the row reduction algorithm and its inverse, backward and forward substitution, approximate substitution, approximate inverse and projection algorithms.

[1]: Chapter 9-11.

Reference:

2. I.N. Herstein and D.J. Winter, *Primer on Linear Algebra*, Macmillan Publishing Company, New York, 1990.
3. S.R. Nagpaul and S.K. Jain, *Topics in Applied Abstract Algebra*, Thomson Brooks and Cole, Belmont, 2005.

SEC-1 LaTeX and HTML

2 Lectures + 2 Practical per week

Elements of LaTeX; Hands-on-training of LaTeX; graphics in LaTeX; PSTricks; Beamer presentation; HTML, creating simple web pages, images and links, design of web pages.

[1] Chapter 9-11, 15

Practical

Six practical should be done by each student. The teacher can assign practical from the exercises from [1].

References:

[1] Martin J. Erickson and Donald Bindner, A Student's Guide to the Study, Practice, and Tools of Modern Mathematics, CRC Press, Boca Raton, FL, 2011.

[2] L. Lamport, LATEX: A Document Preparation System, User's Guide and Reference Manual. Addison-Wesley, New York, second edition, 1994.

SEC-2 Computer Algebra Systems and Related Softwares

2 Lectures + 2 Practical per week

Use of Mathematica, Maple, and Maxima as calculator, in computing functions, in making graphs; MATLAB/Octave for exploring linear algebra and to plot curve and surfaces; the statistical software R: R as a calculator, explore data and relations, testing hypotheses, generate table values and simulate data, plotting.

[1] Chapter 12-14

Practical

Six practical should be done by each student. The teacher can assign practical from the exercises from [1].

References:

[1] Martin J. Erickson and Donald Bindner, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*, CRC Press, Boca Raton, FL, 2011.

S. No.	Existing	Proposed
1	Skill Enhancement Courses (SEC) offered to B.Sc. (H) Mathematics (in 3 rd and 4 th Semester) are of 3 Credits	Since these courses should of 4 Credits according to UGC Guidelines, amendments have been made in the existing Courses to make them of 4 Credits
2	Skill Enhancement Courses (SEC) offered to B.A./ B.Sc. Programme (in 3 rd , 4 th , 5 th and 6 th Semesters) are of 3 Credits	Since these courses should of 4 Credits according to UGC Guidelines, amendments have been made in the existing Courses to make them of 4 Credits
3	Only One Generic Elective Paper is offered (in 1 st , 2 nd and 3 rd Semester) to students of B.Sc. (H), B.A. (H) & B.Com (H) other than B.Sc. (H) Mathematics.	Two Generic Elective Papers are now offered each semester to students of B.Sc. (H), B.A. (H) & B.Com (H) other than B.Sc. (H) Mathematics.
4	No Generic Elective papers were being offered to students of B.A, B.Sc. & B.Com Programme in the 5 th and 6 th Semester	Generic Elective papers are now offered to students of B.A, B.Sc. & B.Com Programme in the 5 th and 6 th Semester

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**SKILL ENHANCEMENT COURSES (SEC)
IN
B.SC. (H) MATHEMATICS**

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC) (2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE) (4)	Generic Elective (GE) (4)
I	C1	(English/ Hindi/ MIL Communication) OR			GE-1
	C2	Environmental Science			
II	C3	Environmental Science OR			GE-2
	C4	(English/ Hindi/ MIL Communication)			
III	C5		SEC-1 (LaTeX and HTML)		GE-3
	C6				
	C7				
IV	C8		SEC-2 (Computer Algebra Systems and Related Softwares)		GE-4
	C9				
	C10				
V	C11			DSE-1	
	C12			DSE-2	
VI	C13			DSE-3	
	C14			DSE-4	

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SEC-1: LaTeX and HTML

4 Credits (2 Lectures + 4 Practical per week)

Theory: 50 marks (including internal assessment)

Practical: 50 marks

UNIT-I

Introduction to TeX and LaTeX, typesetting a simple document, adding basic information to a document, environments, footnotes, sectioning and displayed material

Assents and symbols, Mathematical Typesetting (Elementary and Advanced): subscript/superscript, fractions, roots, ellipsis, mathematical symbols, arrays, delimiters, multiline formulas, spacing and changing style in math mode

Graphics in LaTeX, simple pictures using PS Tricks, Plotting of functions

Beamer Presentation

[1] Chapter 9 (9.1-9.8), Chapter 10 (10.1-10.3), Chapter 11 (11.1-11.4)

[2] Chapter 2 (2.1-2.5), Chapter 3 (3.1-3.3), Chapter 7 (7.1-7.2)

UNIT-II

HTML, creating simple web pages, images and links, design of web pages

[1] Chapter 15 (15.1-15.5)

Practical

(Ideal Lab Practical Batch Size: 15-20 Students)

[1] Chapter 9 (Exercises 4-10), Chapter 10 (Exercises 1, 3, 4, 6-9), Chapter 11 (Exercises 1, 3, 4, 5), Chapter 15 (Exercises 5, 6, 8, 9, 10, 11)

References:

[1] Martin J. Erickson and Donald Bindner, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*, CRC Press, Boca Raton, FL, 2011.

[2] L. Lamport, *LATEX: A Document Preparation System, User's Guide and Reference Manual*. Addison-Wesley, New York, 2nd edition, 1994.

SEC-2: Computer Algebra Systems and Related Softwares

4 Credits (2 Lectures + 4 Practical per week)

Theory: 50 marks (including internal assessment)

Practical: 50 marks

UNIT-I

Computer Algebra Systems (CAS), use of a CAS as a calculator

Computing and plotting functions in 2D, Plotting functions of two variables using Plot3D and ContourPlot, plotting parametric curves and surfaces, Customizing Plots, Animating plots, producing table of values, working with piecewise defined functions, combining graphics

Simple Programming in a CAS

Working with matrices, performing gauss elimination, operations (transpose, determinant, inverse), minors and cofactors, working with large matrices, solving system of linear equations, rank and nullity of a matrix, eigenvalue, eigenvector and diagonalization

[1] Chapter 12 (12.1-12.5)

[2] Chapter 1, Chapter 3 (3.1-3.6, 3.8), Chapter 6 (6.2, 6.3), Chapter 7 (7.1-7.8)

Note: Theoretical and Practical demonstration should be carried out only in one of the CAS: Maxima/ Mathematica/ Maple or any other.

UNIT-II

Statistical software R, R as a calculator, reading and getting data into R: combine and scan commands, types and structure of data items with their properties, Manipulating vectors, data frames, matrices & lists, viewing objects within objects, constructing data objects & conversions, Summary commands, Summary statistics for vectors, data frames, matrices & lists, summary tables, Stem & leaf Plot, Histogram, Plotting in R: Box-whisker Plots, Scatter Plot, Pairs Plot, line charts, Pie Chart, Cleveland Dot Charts, Bar Charts, explore data & relations, saving graphs

[1] Chapter 14 (14.1-14.4, 14.7)

[2] Chapters 2-5, 7

Practical

(Ideal Lab Practical Batch Size: 15-20 Students)

[1] Chapter 12 (Exercises 1-4, 8-12), Chapter 14 (Exercises 1-3)

[2] Chapter 3 [Exercises 3.2 (1), 3.3 (1, 2, 4), 3.4 (1, 2), 3.5 (1-4), 3.6 (2, 3)], Chapter 6 [Exercises 6.2, 6.3], Chapter 7 [Exercises 7.1 (1), 7.2, 7.3 (2), 7.4 (1), 7.5, 7.6]

[3] Relevant exercises of Chapters 2-5, 7 (The practical may be done on the database to be downloaded from <https://data.gov.in/>)

References:

[1] Martin J. Erickson and Donald Bindner, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*, CRC Press, Boca Raton, FL, 2011.

- [2] Bruce F. Torrence and Eve A. Torrence, *The Student's Introduction to MATHEMATICA: A Handbook for Precalculus, Calculus, and Linear Algebra*, Cambridge University Press, 2009.
- [3] M. Gardener, *Beginning R: The Statistical Programming Language*, Wiley Publications, 2012.



S. No.	Existing	Proposed
1	Skill Enhancement Courses (SEC) offered to B.Sc. (H) Mathematics (in 3 rd and 4 th Semester) are of 3 Credits	Since these courses should of 4 Credits according to UGC Guidelines, amendments have been made in the existing Courses to make them of 4 Credits
2	Skill Enhancement Courses (SEC) offered to B.A./ B.Sc. Programme (in 3 rd , 4 th , 5 th and 6 th Semesters) are of 3 Credits	Since these courses should of 4 Credits according to UGC Guidelines, amendments have been made in the existing Courses to make them of 4 Credits
3	Only One Generic Elective Paper is offered (in 1 st , 2 nd and 3 rd Semester) to students of B.Sc. (H), B.A. (H) & B.Com (H) other than B.Sc. (H) Mathematics.	Two Generic Elective Papers are now offered each semester to students of B.Sc. (H), B.A. (H) & B.Com (H) other than B.Sc. (H) Mathematics.
4	No Generic Elective papers were being offered to students of B.A, B.Sc. & B.Com Programme in the 5 th and 6 th Semester	Generic Elective papers are now offered to students of B.A, B.Sc. & B.Com Programme in the 5 th and 6 th Semester

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**GENERIC ELECTIVE COURSES
OFFERED TO
B.SC. (H)/ B. A. (H)/ B. Com (H)
OTHER THAN
B.SC. (H) MATHEMATICS**

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC) (2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE) (4)	Generic Elective (GE) (4)
I					GE-1 Calculus OR Analytic Geometry and Theory of Equations
II					GE-2 Linear Algebra OR Discrete Mathematics
III					GE-3 Differential Equations OR Linear Programming and Game Theory
IV					GE-4 Numerical Methods OR Elements of Analysis
V					
VI					

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Semester I

GE- 1: CALCULUS

OR

GE- 1: ANALYTIC GEOMETRY AND THEORY OF EQUATIONS

GE- 1: CALCULUS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

ϵ - δ Definition of limit of a function, One sided limit, Limits at infinity, Horizontal asymptotes, Infinite limits, Vertical asymptotes, Linearization, Differential of a function, Concavity, Points of inflection, Curve sketching, Indeterminate forms, L'Hopital's rule, Volumes by slicing, Volumes of solids of revolution by the disk method.

UNIT-II

Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves, Area of surface of revolution, Improper integration: Type I and II, Tests of convergence and divergence, Polar coordinates, Graphing in polar coordinates, Vector valued functions: Limit, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector.

UNIT-III

Curvature, Unit normal vector, Torsion, Unit binormal vector, Functions of several Variables, Graph, Level curves, Limit, Continuity, Partial derivatives, Differentiability Chain Rule, Directional derivatives, Gradient, Tangent plane and normal line, Extreme values, Saddle points

REFERENCES:

[1] G. B. Thomas and R. L. Finney, Calculus, Pearson Education, 11/e (2012)

[2] H. Anton, I. Bivens and S. Davis, Calculus, John Wiley and Sons Inc., 7/e (2011)

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OR

GE- 1: ANALYTIC GEOMETRY AND THEORY OF EQUATIONS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

Conic sections and quadratic equations: circle, parabola, ellipse, hyperbola, Techniques for sketching parabola, ellipse and hyperbola, Reflection properties of parabola, ellipse and hyperbola, classifying conic sections by eccentricity, Classification of quadratic equations representing lines, parabola, ellipse and hyperbola. Parameterization of plane curves, conic sections in polar coordinates and their sketching

Unit-II

Rectangular coordinates in 3-space, spheres and cylindrical surfaces, Vectors viewed geometrically, vectors in coordinate system, vectors determine by length and angle, dot product, cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space

UNIT-III

General properties of polynomials and equations, Descarte's rule of signs-positive, negative and imaginary roots, Relation between the roots and the coefficients of equations, Applications, depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by given quantity), removal of terms, Graphical representation of derived function, Rolle's theorem, multiple roots of the equation.

REFERENCES:

- [1] George B. Thomas, Jr., Ross L. Finney, *Calculus and Analytic Geometry*, Pearson Education (Singapore), 2001.
- [2] H. Anton, I. Bivens and S. Davis, *Calculus*, John Wiley and Sons, 2002.
- [3] W.S. Burnside and A.W. Panton, *The Theory of Equations*, Dublin University Press, 1954.

Semester II

GE- 2: LINEAR ALGEBRA

OR

GE- 2: DISCRETE MATHEMATICS

GE- 2: LINEAR ALGEBRA

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

Fundamental operation with vectors in Euclidean space \mathbf{R}^n , Linear combination of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Projection vectors, Some elementary results on vector in \mathbf{R}^n , Matrices, Gauss-Jordan row reduction, Reduced row echelon form, Row equivalence, Rank, Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices, Definition and examples of vector space, Some elementary properties of vector spaces, Subspace.

UNIT-II

Span of a set, A spanning set for an eigenspace, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets, Application of rank, Homogenous and nonhomogenous systems of equations, Coordinates of a vector in ordered basis, Transition matrix, Linear transformations: Definition and examples, Elementary properties, The matrix of a linear transformation, Linear operator and Similarity.

UNIT-III

Application: Computer graphics- Fundamental movements in a plane, Homogenous coordinates, Composition of movements, Kernel and range of a linear transformation, Dimension theorem, One to one and onto linear transformations, Invertible linear transformations, Isomorphism: Isomorphic vector spaces (to \mathbf{R}^n), Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem (Statement only), Orthogonal projection onto a subspace, Application: Least square solutions for inconsistent systems.

REFERENCES:

- [1] S. Andrilli and D. Hecker, Elementary Linear Algebra, Academic Press, 4/e (2012)
- [2] B. Kolman and D.R. Hill, Introductory Linear Algebra with Applications, Pearson Education, 7/e (2003)

OR

GE- 2: DISCRETE MATHEMATICS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

Compound Statements (and, or, implication, negation, contrapositive, quantifiers), truth tables, basic logical equivalences and its consequences, logical arguments, Set theory, operation on sets, types of binary relations, equivalence relation, partial and total ordering, lattices, properties of integers, division algorithm, divisibility and Euclidean algorithm, gcd, lcm, relatively prime, prime numbers, statement of fundamental theorem of arithmetic, Fermat primes,

UNIT-II

Congruences and its properties, Mathematical induction, recursive relations and its solution (characteristics polynomial and generating function), principles of counting (inclusion/exclusion, pigeon-hole), permutation and combinations (with and without repetition),

UNIT-III

Duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms, distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

REFERENCES:

- [1]. B A. Davey and H. A. Priestley, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1990.
- [2]. Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint 2003.
- [3]. Rudolf Lidl and Günter Pilz, *Applied Abstract Algebra* (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
- [4]. Kenneth H Rosen, *Discrete Mathematics and its applications* (7th Edition), Tata McGrawHill Education Private Limited, 2011.

Semester III

GE- 3: DIFFERENTIAL EQUATIONS

OR

GE- 3: LINEAR PROGRAMMING AND GAME THEORY

GE- 3: DIFFERENTIAL EQUATIONS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

First order ordinary differential equations: Basic concepts and ideas, Exact differential equations, Integrating factors, Bernoulli equations, Orthogonal trajectories of curves, Existence and uniqueness of solutions, Second order differential equations: Homogenous linear equations of second order, Second order homogenous equations with constant coefficients, Differential operator, Euler-Cauchy equation.

UNIT-II

Existence and uniqueness theory, Wronskian, Non-homogenous ordinary differential equations, Solution by undetermined coefficients, Solution by variation of parameters, Higher order homogenous equations with constant coefficients, System of differential equations, System of differential equations, Conversion of n th order ODEs to a system, Basic concepts and ideas, Homogenous system with constant coefficients.

UNIT-III

Power series method: Theory of power series methods, Legendre's equation, Legendre polynomial, Partial differential equations: Basic Concepts and definitions, Mathematical problems, First order equations: Classification, Construction, Geometrical interpretation, Method of characteristics, General solutions of first order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations, Classification of second order partial differential equations, Reduction to canonical forms, Second order partial differential equations with constant coefficients, General solutions.

REFERENCES:

- [1] Erwin Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons, Inc., 9/e, (2006)
- [2] TynMyint-U and Lokenath Debnath; Linear Partial Differential Equations for Scientists and Engineers, Springer, Indian Reprint (2009)

OR

GE- 3: LINEAR PROGRAMMING AND GAME THEORY

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

Introduction to linear programming problem, Graphical method of solution, Basic feasible solutions, linear programming and Convexity, Introduction to the Simplex method, Theory of the Simplex method, Optimality and Unboundedness, The simplex tableau and examples, Artificial variables, Introduction to duality, Formulation of the dual problem and its examples, Duality Theorem and its applications

UNIT-II

Transportation problem and its mathematical formulation, Northwest-corner method, Least-cost method and Vogel approximation method for determination of starting basic solution, Algorithm for solving transportation problem, Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem

UNIT-III

Introduction to Game theory, Formulation of two-person zero sum rectangular game, Solution of rectangular games with saddle points, dominance principle; rectangular games without saddle point –mixed strategy, Graphical, algebraic and linear programming solution of $m \times n$ games

REFERENCES:

- [1]. Paul R. Thie and Gerard E. Keough, *An Introduction to Linear Programming and Game Theory*, Third Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2008.
- [2]. Hamdy A. Taha, *Operations Research: An Introduction*, Ninth Edition, Prentice Hall, 2011.
- [3]. Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, Ninth Edition, McGraw-Hill, Inc., New York, 2010.

Semester IV

GE- 4: NUMERICAL METHODS

OR

GE- 4: ELEMENTS OF ANALYSIS

GE- 4: NUMERICAL METHODS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT-I

Floating point representation and computer arithmetic, Significant digits, Errors: Roundoff error, Local truncation error, Global truncation error, Order of a method, Convergence and terminal conditions, Efficient computations Bisection method, Secant method, Regula-Falsi method, Newton Raphson method, Newton's method for solving nonlinear systems

UNIT-II

Gauss elimination method (with row pivoting) and Gauss-Jordan method, Gauss Thomas method for tridiagonal systems Iterative methods: Jacobi and Gauss-Seidel iterative methods Interpolation: Lagrange's form and Newton's form Finite difference operators, Gregory Newton forward and backward differences Interpolation.

UNIT-III

Piecewise polynomial interpolation: Linear interpolation, Cubic spline interpolation (only method), Numerical differentiation: First derivatives and second order derivatives, Richardson extrapolation Numerical integration: Trapezoid rule, Simpson's rule (only method), Newton-Cotes open formulas, Extrapolation methods: Romberg integration, Gaussian quadrature, Ordinary differential equation: Euler's method Modified Euler's methods: Heun method and Mid-point method, Runge-Kutta second methods: Heun method without iteration, Mid-point method and Ralston's method Classical 4th order Runge-Kutta method, Finite difference method for linear ODE

REFERENCES:

- [1] Laurence V. Fausett, Applied Numerical Analysis, Using MATLAB, Pearson, 2/e (2012)
- [2] M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, 6/e (2012)
- [3] Steven C Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientists, Tata McGraw Hill, 2/e (2010)

OR

GE- 4: ELEMENTS OF ANALYSIS

5 Lectures + 1 Tutorial per week (Ideal Tutorial Group Size: 12-15 Students)

Max. Marks: 100 (including internal assessment)

Examination: 3 hrs

UNIT I

Finite and infinite sets examples of countable and uncountable sets. Real line; absolute value bounded sets suprema and infima, statement of order Completeness property of \mathbb{R} , Archimedean property of \mathbb{R} , intervals. Real sequences, Convergence, sum and product of convergent sequences, proof of convergence of some simple sequences such as $(-1)^n/n$, $1/n^2$, $(1+1/n)^n$, x^n with $|x| < 1$, a_n/n , where a_n is a bounded sequence, Concept of cluster points and statement of Bolzano Weierstrass' theorem, Statement and illustration of Cauchy convergence criterion for sequences, Cauchy's theorem on limits, order preservation and squeeze theorem, monotone sequences and their convergence

UNIT II

Definition and a necessary condition for convergence of an infinite series, Cauchy convergence criterion for series, positive term series, geometric series, comparison test, limit comparison test, convergence of p-series, Root test, Ratio test, alternating series, Leibnitz's test. Definition and examples of absolute and conditional convergence

UNIT III

Definition of power series: radius of convergence, Cauchy-Hadamard theorem, statement and illustration of term-by-term differentiation and integration of power series, Power series expansions for $\exp(x)$, $\sin(x)$, $\cos(x)$, $\log(1+x)$ and their properties

REFERENCES:

- [1] R.G. Bartle and D.R. Sherbert: *Introduction to Real Analysis*, John Wiley and Sons (Asia) Pte. Ltd., 2000.
- [2] C. P. Simon and L. Blume: *Mathematics for Economists*, W W Norton and Company, 1994.
- [3] K. Sydsaeter and P.J. Hammod, *Mathematics for Economics Analysis*, Pearson Education, 2002

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