#### DEPARTMENT OF MATHEMATICS

# COURSES & SYLLABI OF SEMESTER – III B.Sc. (Hons) MATHEMATICS

Category-I

#### DISCIPLINE SPECIFIC CORE COURSE -7: GROUP THEORY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	listribution	of the course	Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Group	4	3	1	0	Class XII	DSC-I:
Theory					pass with	Algebra
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

- Symmetric groups, normal subgroups, factor groups, and direct products of groups.
- The notions of group homomorphism to study the isomorphism theorems with applications.
- Classification of groups with small order according to isomorphisms.

# **Learning Outcomes**

This course will enable the students to:

- Analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons.
- Understand the significance of the notion of cosets, Lagrange's theorem and its consequences.
- Know about group homomorphisms and isomorphisms and to relate groups using these mappings.
- Express a finite abelian group as the direct product of cyclic groups of prime power orders.
- Learn about external direct products and its applications to data security and electric circuits.

#### **SYLLABUS OF DSC - 7**

#### Unit - 1 (18 hours)

#### Permutation Groups, Lagrange's Theorem and Normal Subgroups

Permutation groups and group of symmetries, Cycle notation for permutations and properties, Even and odd permutations, Alternating groups; Cosets and its properties, Lagrange's theorem and consequences including Fermat's Little theorem, Number of elements in product of two finite subgroups; Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups.

# Unit - 2 (15 hours)

#### **Group Homomorphisms and Automorphisms**

Group homomorphisms, isomorphisms and properties, Cayley's theorem; First, Second and Third isomorphism theorems for groups; Automorphism, Inner automorphism, Automorphism groups, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.

Unit - 3 (12 hours)

# **Direct Products of Groups and Fundamental Theorem of Finite Abelian Groups**

External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits; Internal direct products; Fundamental theorem of finite abelian groups and its isomorphism classes.

#### **Essential Reading**

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.

#### **Suggestive Readings**

- Artin, Michael. (1991). Algebra (2nd ed.). Pearson Education. Indian Reprint 2015.
- Dummit, David S., & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India, Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

# DISCIPLINE SPECIFIC CORE COURSE -8: RIEMANN INTEGRATION

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit distribution of the course			Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Riemann	4	3	1	0	Class XII	DSC-2 and 5:
Integration					pass with	Elementary Real
					Mathematics	Analysis, and
						Calculus

#### **Learning Objectives**

The primary objective of this course is to:

- Understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration.
- Learn some of the properties of Riemann integrable functions, its generalization and the applications of the fundamental theorems of integration.
- Get an exposure to the utility of integration for practical purposes.

#### **Learning Outcomes**

This course will enable the students to:

- Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Riemann sums to the volume and surface of a solid of revolution.
- Get insight of integration by substitution and integration by parts.
- Know about convergence of improper integrals including, beta and gamma functions.

Unit - 1 (18 hours)

#### The Riemann Integral

Definition of upper and lower Darboux sums, Darboux integral, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Darboux integrability; Riemann's definition of integrability by Riemann sum and the equivalence of Riemann's and Darboux's definitions of integrability; Definition and examples of the Riemann-Stieltjes integral.

Unit - 2 (15 hours)

#### **Properties of The Riemann Integral and Fundamental Theorems**

Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions; Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability; Intermediate value theorem for integrals, Fundamental Theorems of Calculus (I and II).

Unit - 3 (12 hours)

#### **Applications of Integrals and Improper Integrals**

Methods of integration: integration by substitution and integration by parts; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution. Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, The beta and gamma functions and their properties.

#### **Essential Readings**

- 1. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.
- 2. Anton, Howard, Bivens Irl and Davis Stephens (2012). Calculus (10th edn.). John Wiley & Sons, Inc.
- 3. Denlinger, Charles G. (2011). Elements of Real Analysis, Jones & Bartlett India Pvt. Ltd., Indian Reprint.
- 4. Ghorpade, Sudhir R. and Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). Indian Reprint.

#### **Suggestive Readings**

- Bartle, Robert G., & Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley, Indian Edition.
- Kumar Ajit and Kumaresan S. (2014). A Basic Course in Real Analysis. CRC Press, Taylor & Francis Group, Special Indian Edition.

# DISCIPLINE SPECIFIC CORE COURSE-9: DISCRETE MATHEMATICS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

<b>Course title</b>	Credits	Credit distribution of the course			Eligibility	Pre-requisite
& Code		Lecture	Tutorial	Practical/	criteria	of the course
				Practice		(if any)
Discrete					Class XII	DSC-I and IV:
Mathematics	4	3	0	1	pass with	Algebra and
					Mathematics	Linear Algebra

#### **Learning Objectives**

The primary objective of the course is to:

- Make students embark upon a journey of enlightenment, starting from the abstract concepts in mathematics to practical applications of those concepts in real life.
- Make the students familiar with the notion of partially ordered set and a level up with the study of lattice, Boolean algebra and related concepts.
- Culminate the journey of learning with practical applications using the knowledge attained from the abstract concepts learnt in the course.

#### **Learning Outcomes**

This course will enable the students to:

- Understand the notion of partially ordered set, lattice, Boolean algebra with applications.
- Handle the practical aspect of minimization of switching circuits to a great extent with the methods discussed in this course.
- Apply the knowledge of Boolean algebras to logic, set theory and probability theory.

#### **SYLLABUS OF DSC - 9**

# Unit - 1 (15 hours)

#### **Cardinality and Partially Ordered Sets**

The cardinality of a set; Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams, Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

**Unit – 2** (15 hours)

#### Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism; Definitions, examples and properties of modular and distributive lattices; The  $M_3-N_5$  theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit - 3 (15 hours)

#### **Boolean Algebras and Applications**

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials, Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory.

#### **Practical (30 hours):**

Practical/Lab work to be performed in a computer Lab using any of the Computer Algebra System Software such as Mathematica/MATLAB /Maple/Maxima/Scilab/SageMath etc., for the following problems based on:

- 1) Expressing relations as ordered pairs and creating relations.
- 2) Finding whether or not, a given relation is:
  - i. Reflexive ii. Antisymmetric iii. Transitive iv. Partial order
- 3) Finding the following for a given partially ordered set
  - i. Covering relations.
  - ii. The corresponding Hasse diagram representation.
  - iii. Minimal and maximal elements.
- 4) Finding the following for a subset *S* of a given partially ordered set *P* 
  - i. Whether a given element in *P* is an upper bound (lower bound) of *S* or not.
  - ii. Set of all upper bounds (lower bounds) of *S*.
  - iii. The least upper bound (greatest lower bound) of S, if it exists.
- 5) Creating lattices and determining whether or not, a given partially ordered set is a lattice.
- 6) Finding the following for a given Boolean polynomial function:
  - i. Representation of Boolean polynomial function and finding its value when the Boolean variables in it take particular values over the Boolean algebra {0,1}.
  - ii. Display in table form of all possible values of Boolean polynomial function over the Boolean algebra {0,1}.
- 7) Finding the following:
  - i. Dual of a given Boolean polynomial/expression.
  - ii. Whether or not two given Boolean polynomials are equivalent.
  - iii. Disjunctive normal form (Conjunctive normal form) from a given Boolean expression.
  - iv. Disjunctive normal form (Conjunctive normal form) when the given Boolean polynomial function is expressed by a table of values.
- 8) Representing a given circuit diagram (expressed using gates) in the form of Boolean expression.
- 9) Minimizing a given Boolean expression to find minimal expressions.

#### **Essential Readings**

- 1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
- 2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

#### **Suggested Readings**

- Donnellan, Thomas. (1999). Lattice Theory (1st ed.). Khosla Pub. House. Indian Reprint.
- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.

# **B.Sc.** (Hons) Mathematics, Semester-III, DSE-Courses

#### DISCIPLINE SPECIFIC ELECTIVE COURSE -1(i): GRAPH THEORY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
title &		Lecture Tutorial Practical/			criteria	of the course
Code				Practice		(if any)
Graph	4	3	1	0	Class XII	Nil
Theory					pass with	
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

#### **Learning Outcomes**

This course will enable the students to:

- Learn modelling of real-world problems by graphs.
- Know characteristics of different classes of graphs.
- Learn representation of graphs in terms of matrices.
- Learn algorithms to optimize a solution.
- Understand some properties of graphs and their applications in different practical situations.

#### **SYLLABUS OF DSE - 1(i)**

Unit - 1 (12 hours)

#### Graphs, Paths and Circuits

Definition, Examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Connected graphs, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm.

Unit - 2 (15 hours)

#### **Applications of Paths and Circuits, Trees**

Applications of Path and Circuits: The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm, Tournaments, Scheduling Problem, Trees, Properties of Trees, Spanning Trees, Minimum Spanning Tree Algorithms.

Unit - 3 (18 hours)

# **Connectivity and Graph Coloring, Planar Graphs**

Cut-vertices, Blocks and their Characterization, Connectivity and edge-connectivity, Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring and applications, Matchings, Hall's theorem, Independent sets and covers.

#### **Essential Readings**

- 1. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
- 2. Chartrand, Gary, & Zhang, Ping (2012). A First Course in Graph Theory. Dover Publications.

#### **Suggestive Readings**

- Bondy, J. A., and Murty, U.S.R. (2008). Graph Theory. Graduate Texts in Mathematics, Springer.
- Diestel, Reinhard (2017). Graph Theory (5th ed.). Graduate Texts in Mathematics, Springer.
- West, Douglas B. (2001). Introduction to Graph Theory (2nd ed.). Prentice Hall. Indian Reprint.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

# DISCIPLINE SPECIFIC ELECTIVE COURSE– 1(ii): MATHEMATICAL PYTHON

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title	Credits	Credit distribution of the course			Eligibility	Pre-requisite
& Code		Lecture	Tutorial	Practical/	criteria	of the course
				Practice		(if any)
Mathematical					Class XII	Basic
Python	4	3	0	1	pass with	knowledge
					Mathematics	of Python

#### **Learning Objectives**

The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

#### **Learning Outcomes**

This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

#### **SYLLABUS OF DSE - 1(ii)**

#### **Theory**

Unit - 1 (15 hours)

#### **Drawing Shapes, Graphing and Visualization**

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

Unit - 2 (18 hours)

#### **Numerical and Symbolic Solutions of Mathematical Problems**

NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

Unit - 3 (12 hours)

#### **Document Generation with Python and LaTeX**

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

#### **Essential Readings**

- 1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
- 2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
- 3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

#### **Suggested Readings**

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
  - o https://matplotlib.org
  - o https://sympy.org
  - o https://pandas.pydata.org
  - o https://numpy.org
  - o https://pypi.org
  - o https://patrickwalls.github.io/mathematicalpython/

#### DISCIPLINE SPECIFIC ELECTIVE COURSE-1(iii): NUMBER THEORY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	listribution	of the course	Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Number	4	3	1	0	Class XII	DSC-I:
Theory					pass with	Algebra
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

- The number theoretic techniques of computations with the flavour of abstraction.
- The Euclidean algorithm, linear Diophantine equations, congruence equations, arithmetic functions and their applications, Fermat's little, Euler's and Wilson's theorems.
- Primitive roots, quadratic residues and nonresidues, the Legendre symbol and the law of Quadratic Reciprocity.
- Introduction to cryptography, public-key cryptosystems and applications.

#### **Learning Outcomes**

This course will enable the students to:

- Use modular arithmetic in solving linear and system of linear congruence equations.
- Work with the number theoretic functions, their properties and their use.
- Learn the forms of positive integers that possess primitive roots and the Quadratic Reciprocity Law which deals with the solvability of quadratic congruences.
- Understand the public-key cryptosystems, in particular, RSA.

#### **SYLLABUS OF DSE - 1(iii)**

#### Unit - 1 (12 hours)

#### **Linear Diophantine equation and Theory of Congruences**

The Euclidean Algorithm and linear Diophantine equation; Least non-negative residues and complete set of residues modulo n; Linear congruences, The Chinese remainder theorem and system of linear congruences in two variables; Fermat's little theorem, Wilson's theorem and its converse, Application to solve quadratic congruence equation modulo odd prime p.

#### Unit -2 (21 hours)

#### **Number-Theoretic Functions and Primitive Roots**

Number-theoretic functions for the sum and number of divisors, Multiplicative function, Möbius inversion formula and its properties; Greatest integer function with an application to the calendar; Euler's Phi-function, Euler's theorem and some properties of the Phi-function; The order of an integer modulo n and primitive roots for primes, Primitive roots of composite numbers n: when n is of the form  $2^k$ , and when n is a product of two coprime numbers.

Unit - 3 (12 hours)

# **Quadratic Reciprocity Law and Public Key Cryptosystems**

The quadratic residue and nonresidue of an odd prime and Euler's criterion, The Legendre symbol and its properties, Quadratic Reciprocity law and its application; Introduction to cryptography, Hill's cipher, Public-key cryptography and RSA.

#### **Essential Reading**

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

#### **Suggestive Readings**

- Andrews, George E. (1994). Number Theory. Dover publications, Inc. New York.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

# B.A. (Prog.) with Mathematics as Major

# **Category-II**

# DISCIPLINE SPECIFIC CORE COURSE – 3: THEORY OF EOUATIONS AND SYMMETRIES

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit di	istribution	of the course	Eligibility	Pre-requisite
title & Code		Lecture	Tutorial	Practical/ Practice	criteria	of the course (if any)
Theory of Equations and Symmetries	4	3	1	0	Class X pass with Mathematics	Nil

#### **Learning Objectives**

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

#### **Learning Outcomes**

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

#### **SYLLABUS OF DSC-3**

# Unit - 1 (18 hours)

#### **Polynomial Equations and Properties**

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions.

#### Unit - 2 (12 hours)

#### **Cubic and Biquadratic (Quartic) Equations**

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit - 3 (15 hours)

#### **Symmetric Functions**

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as  $\sum \alpha^2 \beta$ ,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ ,... of

polynomial equations; Transformation of equations by symmetric functions and in general.

#### **Essential Readings**

- 1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
- 2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

#### **Suggestive Readings**

• Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

# DISCIPLINE SPECIFIC CORE COURSE – A-3: DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	<b>Credit distribution of the course</b>			Pre-requisite
title &		Lecture Tutorial Practical/			criteria	of the course
Code				Practice		(if any)
Differential					Class XII	
Equations	4	3	1	0	pass with	Nil
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

#### **Learning Outcomes**

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and system of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

Unit - 1 (15 hours)

#### **Ordinary Differential Equations**

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

Unit - 2 (12 hours)

#### **Explicit Methods of Solving Higher-Order Linear Differential Equations**

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

Unit - 3 (18 hours)

#### First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### **Essential Readings**

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### **Suggestive Readings**

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

# B.A./B.Sc. (Prog.) with Mathematics as Non-Major

# **Category-III**

# DISCIPLINE SPECIFIC CORE COURSE – A-3: DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
title &		Lecture Tutorial Practical/		criteria	of the course	
Code				Practice		(if any)
Differential					Class XII	
Equations	4	3	1	0	pass with	Nil
					Mathematics	

### **Learning Objectives**

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

# **Learning Outcomes**

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and System of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

#### **SYLLABUS of Discipline A-3**

# Unit - 1 (15 hours)

# **Ordinary Differential Equations**

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

# Unit - 2 (12 hours)

#### **Explicit Methods of Solving Higher-Order Linear Differential Equations**

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

Unit - 3 (18 hours)

#### First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### **Essential Readings**

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### **Suggestive Readings**

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

# B.Sc. (Physical Sciences/Mathematical Sciences) with Mathematics as one of the Core Discipline

# **Category-III**

# DISCIPLINE SPECIFIC CORE COURSE – A-3: DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	istribution	of the course	Eligibility	Pre-requisite
title &		Lecture   Tutorial   Practical/			criteria	of the course
Code				Practice		(if any)
Differential					Class XII	
Equations	4	3	1	0	pass with	Nil
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

#### **Learning Outcomes**

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and System of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

#### **SYLLABUS of Discipline A-3**

# Unit - 1 (15 hours)

#### **Ordinary Differential Equations**

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

# Unit - 2 (12 hours)

#### **Explicit Methods of Solving Higher-Order Linear Differential Equations**

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

Unit - 3 (18 hours)

#### First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### **Essential Readings**

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### **Suggestive Readings**

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-III

**DISCIPLINE SPECIFIC ELECTIVE -1(i): COMBINATORICS** 

# CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title	Credits	Credit di	stribution	of the course	Eligibility	Pre-requisite
& Code		Lecture	Tutorial	Practical/	criteria	of the course
				Practice		(if any)
					Class XII	
Combinatorics	4	3	1	0	pass with	Nil
					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to:

- Introduce various techniques of permutations, combinations and inclusion-exclusion.
- Learn basic models of generating functions and recurrence relations in their application to the theory of integer partitions.

#### **Learning Outcomes**

After completing the course, student will:

- Enhance the mathematical logical skills by learning different enumeration techniques.
- Be able to apply these techniques in solving problems in other areas of mathematics.
- Be trained to provide reasoning and arguments to justify conclusions.

#### **SYLLABUS OF DSE-1(i)**

**Unit - 1** (15 hours)

#### **Basics of Combinatorics**

Basic counting principles, Permutations and Combinations (with and without repetitions), Binomial coefficients, Multinomial coefficients, Counting subsets of size k; Set-partitions, The inclusion-exclusion principle and applications.

**Unit - 2** (18 hours)

#### **Generating Functions and Recurrence Relations**

Generating functions: Generating function models, Calculating coefficients of generating functions, Polynomial expansions, Binomial identity, Exponential generating functions. Recurrence relations: Recurrence relation models, Divide-and-conquer relations, Solution of linear recurrence relations, Solutions by generating functions.

Unit – 3 (12 hours)

#### **Partition**

Partition theory of integers: Ordered partition, Unordered partition, Ferrers diagram, Conjugate of partition, Self-conjugate partition, Durfee square, Euler's pentagonal theorem.

#### **Essential Readings**

- 1. Sane, Sharad S. (2013). Combinatorial Techniques. Hindustan Book Agency (India).
- 2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.

#### **Suggested Readings**

- Brualdi, Richard A. (2009). Introductory Combinatorics (5th ed.). Pearson Education Inc.
- Cameron, Peter J. (1994). Combinatorics: Topics, Techniques, Algorithms. Cambridge University Press.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

# DISCIPLINE SPECIFIC ELECTIVE COURSE-1(ii): ELEMENTS OF NUMBER THEORY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit d	listribution	of the course	Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Elements of					Class XII	
Number	4	3	1	0	pass with	Nil
Theory					Mathematics	

#### **Learning Objectives**

The primary objective of this course is to introduce:

• The Euclidean algorithm and linear Diophantine equations, the Fundamental theorem of arithmetic and some of the open problems of number theory viz. the Goldbach conjecture.

- The modular arithmetic, linear congruence equations, system of linear congruence equations, arithmetic functions and multiplicative functions, e.g., Euler's Phi-function.
- Introduction of the simple encryption and decryption techniques, and the numbers of specific forms viz. Mersenne numbers, Fermat numbers etc.

#### **Learning Outcomes**

This course will enable the students to:

- Get familiar with the basic number-theoretic techniques.
- Comprehend some of the open problems in number theory.
- Learn the properties and use of number-theoretic functions and special types of numbers.
- Acquire knowledge about public-key cryptosystems, particularly RSA.

#### **SYLLABUS OF DSE-1(ii)**

Unit - 1 (12 hours)

#### **Divisibility and Prime Numbers**

Revisiting: The division algorithm, divisibility and the greatest common divisor. Euclid's lemma; The Euclidean algorithm, Linear Diophantine equations; The Fundamental theorem of Arithmetic, The sieve of Eratosthenes, Euclid theorem and the Goldbach conjecture; The Fibonacci sequence and its nature.

Unit - 2 (21 hours)

#### **Theory of Congruences and Number-Theoretic Functions**

Congruence relation and its basic properties, Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables; Fermat's little theorem and its generalization, Wilson's theorem and its converse; Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions, The greatest integer function; Euler's Phi-function and its properties.

Unit - 3 (12 hours)

### **Public Key Encryption and Numbers of Special Form**

Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique; Introduction to perfect numbers, Mersenne numbers and Fermat numbers.

#### **Essential Reading**

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

#### **Suggestive Readings**

- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

# DISCIPLINE SPECIFIC ELECTIVE COURSE - DSE-1(iii): THEORY OF EQUATIONS AND SYMMETRIES

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	<b>Credit distribution of the course</b>			Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Theory of					Class X	
Equations	4	3	1	0	pass with	Nil
and					Mathematics	
Symmetries						

#### **Learning Objectives**

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

# **Learning Outcomes**

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

#### **SYLLABUS OF DSE-1(iii)**

#### Unit - 1 (18 hours)

#### **Polynomial Equations and Properties**

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions.

#### Unit - 2 (12 hours)

#### **Cubic and Biquadratic (Quartic) Equations**

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit - 3 (15 hours)

#### **Symmetric Functions**

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as  $\sum \alpha^2 \beta$ ,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ ,... of polynomial equations; Transformation of equations by symmetric functions and in general.

#### **Essential Readings**

- 1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
- 2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

#### **Suggestive Readings**

• Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

# COMMON POOL OF GENERIC ELECTIVES (GE) COURSES OFFERED BY DEPARTMENT OF MATHEMATICS Category-IV

# GENERIC ELECTIVES-GE-3(i): DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit distribution of the course			Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Differential					Class XII	
Equations	4	3	1	0	pass with	Nil
					Mathematics	

# **Learning Objectives**

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

# **Learning Outcomes**

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and System of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

#### **SYLLABUS OF GE-3(i)**

# Unit - 1 (15 hours)

#### **Ordinary Differential Equations**

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

#### Unit - 2 (12 hours)

#### **Explicit Methods of Solving Higher-Order Linear Differential Equations**

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

Unit - 3 (18 hours)

#### First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### **Essential Readings**

- 1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
- 2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### **Suggestive Readings**

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

GENERIC ELECTIVES-GE-3(ii): LATTICES AND NUMBER THEORY

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course	Credits	Credit distribution of the course			Eligibility	Pre-requisite
title &		Lecture	Tutorial	Practical/	criteria	of the course
Code				Practice		(if any)
Lattices					Class XII	
and	4	3	1	0	pass with	Nil
Number					Mathematics	
Theory						

#### **Learning Objectives**

The primary objective of this course is to introduce:

- The concepts of ordered sets, lattices, sublattices and homomorphisms between lattices.
- Distributive lattices along with Boolean algebra and their applications in the real-world.
- Divisibility theory of congruences along with some applications.
- The number-theoretic functions and quadratic reciprocity law.

#### **Learning Outcomes**

This course will enable the students to:

- Understand the notion of ordered sets. Learn about lattices, distributive lattices, sublattices and homomorphisms between lattices.
- Become familiar with Boolean algebra, Boolean polynomials, switching circuits and their applications.
- Learn the concept of Karnaugh diagrams and Quinn–McCluskey method which gives an aid to apply truth tables in real-world problems.

- Learn about some fascinating properties of prime numbers, and some of the open problems in number theory, viz., Goldbach conjecture etc.
- Know about modular arithmetic and number-theoretic functions like Euler's Phi-function.
- Find quadratic residues and nonresidues modulo primes using Gauss's Quadratic Reciprocity Law.

#### **SYLLABUS OF GE-3(ii)**

Unit - 1 (21 hours)

#### **Partially Ordered Sets and Lattices**

Definitions, Examples and basic properties of partially ordered sets, Order isomorphism, Hasse Diagram, Maximal and minimal elements, Dual of an ordered set, Duality principle; Statements of Well Ordering Principle and Zorn's Lemma; Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications.

Unit - 2 (12 hours)

### **Divisibility and Theory of Congruences**

The division algorithm: GCD, The Euclidean algorithm, Diophantine equation ax + by = c Primes: The Fundamental Theorem of Arithmetic, Infinitude of primes, Twin primes and Goldbach conjecture.

The theory of congruences: Basic properties and applications, Linear congruences and the Chinese Remainder Theorem, Fermat's Little Theorem and Wilson's Theorem.

Unit - 3 (12 hours)

#### Number-Theoretic Functions, Primitive roots and Quadratic Reciprocity Law

Number-Theoretic Functions: Sum and number of divisors, Euler's Phi-function and Euler's generalization of Fermat's Little Theorem.

Primitive roots: The order of an integer modulo n, and primitive roots of an integer.

Quadratic Reciprocity Law: Quadratic residue and nonresidue, Euler's Criterion, The Legendre symbol and its properties and Quadratic Reciprocity Law.

#### **Essential Readings**

- 1. Davey, B A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.), Cambridge University Press, Cambridge.
- 2. Lidl, Rudolf & Pilz, Günter. (1998). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics, Springer (SIE), Indian Reprint 2004.
- 3. Burton, David M. (2012). Elementary Number Theory (7th ed.), Mc-Graw Hill Education Pvt. Ltd. Indian Reprint.

#### **Suggestive Readings**

- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.
- Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.
- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.