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Your Roll No....3039.....

Sr. No. of Question Paper : 2736

Unique Paper Code : 12481202

Name of the Paper : Mathematics for Business
Economics

Name of the Course : B.A. (Hons.) Business
Economics, 2019 (CBCS)

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer **all** questions.
3. Choice is available within each question.
4. Use of simple calculator is permitted.

1. Attempt any **three** parts from parts (a) to (d) in this question. (5×3)

P.T.O.

(a) For each value of n from 1 to infinity, consider the sets described by the open intervals $(-1 - n^{-1}, 1 + n^{-1})$. Find the interval that represents the intersection of all these sets.

(b) Given the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$$

find the points where $f(x)$ is continuous or discontinuous.

(c) On the same graph, draw the regions represented by the sets :

$$P = \{(x, y) \mid x^2 + y^2 \leq 25\} \text{ and } Q = \{(x, y) \mid xy \geq 12\} \text{ and}$$

plot the coordinates of all the points where the graphs intersect each other or intersect the coordinate axes. Are P and Q disjoint?

(d) Find all asymptotes to the curve represented by the equation :

$$y = \frac{x^3 - 7x^2 + 16x - 12}{x^2 - 4x + 3}$$

2x Attempt any four parts from parts (a) to (e) in this question. (5×4)

(a) (i) If a function is known to be strictly increasing and differentiable, what can we conclude about the sign of the derivative of the function?

(ii) Check whether the function $f(x) = x^4 - 4x^3$ is increasing or decreasing, and has a maximum or minimum at $x = 0$.

(b) Use limits and derivatives of the curve $F(x) = x^2 + \ln(x)$ to comment on its shape and draw a graph of the curve.

(c) A competitive industry faces a demand $x = 865 - 5p$, where x is industry output and p is the price of the product. There are n firms having identical cost functions $C_i = 144 + 5x_i + 4x_i^2$, where x_i is the output of firm i and C_i is its total cost. There is free entry and a large number of potential entrants. Find out the equilibrium output and price for this industry. How many firms will enter the industry?

(d) If the population of a country grows over time according to the function $P(t)$ and the national

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income grows according to the function $\dot{Y}(t)$. Derive an expression for the proportional rate of change of per capita national income $Z(t) = Y(t)/P(t)$ in terms of the proportional rates of change of Y and P .

(e) Find the elasticity of $f(x) = ax^2 + bx$ with respect to x . How does the elasticity change as x increases?

3. Attempt any **three** parts from the parts (a) to (d) in this question. (5×3)

(a) Prove that if A is a symmetric or it is a skew symmetric matrix, AA is a symmetric matrix.

(b) Find the equation of the plane through the point $(-1, 2, -5)$ that is perpendicular to the planes $2x - y + z = 1$ and $x + y - 2z = 3$.

(c) A system of linear simultaneous equations is given by

$$x + 2y = 10; 3x + 6y = p; qx + 8y = 40.$$

(i) Find the conditions on p and q for the system to be consistent with unique or many solutions.

(ii) In each case, specify the degrees of freedom of this system and how many of the equations are superfluous?

(d) A Leontief model with two commodities A and B is given. A unit production of commodity A requires 10 units of labour, 2 units of commodity B and 0.1 units of its own output. A unit production of commodity B requires 6 units of labour, 0.2 units of commodity A and 0.3 units of its own output. If the final demands of commodities A and B are 1000 and 2000 units respectively, find

(i) The equilibrium level of gross output.

(ii) The total requirement of labour in equilibrium.

4. Attempt any five parts from the parts (a) to (f) in this question. (5×5)

(a) Calculate the rate of change in z at $t = 0$ if

$$z = \frac{5t^2 + 3xy}{2w^2y}, \quad x = t^2 + 1, \quad y = \sqrt{t^2 + 1}, \quad w = e^t + 1.$$

(b) Evaluate the elasticity of substitution for the

$$\text{function } z = f(x, y) = \left(\frac{0.2}{x} + \frac{0.8}{y} \right)^{-1}.$$

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- (c) (i) Specify the domain and provide a rough sketch of it for the function

$$f(x, y) = \ln(9 - x^2 - 9y^2).$$

- (ii) Provide a rough sketch of the level curve to the function in (i) above at the height 4.

- (d) (i) State the definition of a homothetic function?

- (ii) Provide reasons to decide whether the following functions f and g are homothetic :

$$f(x_1, x_2, \dots, x_n) = A(\delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} + \dots + \delta_n x_n^{-\rho})^{\frac{\mu}{\rho}}$$

$$g(x_1, x_2) = 2 \log x_1 + 5 \log x_2$$

- (e) $x(a, b) = A\sqrt{ab}$ is a production function for good x using inputs a and b . Use the Lagrangian method to find the amount of the factors required to produce an output \bar{x} at minimum cost, when prices of the inputs are p_a and p_b . Check the second order conditions.

- (f) Find the global extreme points for the function

$$f(x, y) = x^2 y^3 \text{ defined on the set}$$

$$\{(x, y) \mid x \geq 1, y \geq 2, x + y \leq 10\}.$$