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[This question paper contains 6 printed pages.]

**Your Roll No.**.....

**Sr. No. of Question Paper : 7344**

Unique Paper Code : 12481202

Name of the Paper : Mathematics for Business Economics

Name of the Course : **B.A. (Hons.) Business Economics, 2017 (CBCS)**

Semester : II

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions.
3. Use of Simple calculator is allowed.

1. Attempt **any 3** parts. (5×3=15)

(a) Prove that $\sqrt{2}$ is irrational.

P.T.O.

(b) If the function $f:[1,\infty]\to[1,\infty]$ is defined by $f(x)=2^{x(x-1)}$ then prove that f is one-one and onto. Hence find the inverse of f.

(c) What is the present value of 15 annual deposits of Rs. 3500 each when the first deposit is 1 year from now and the interest rate is 12% per annum.

(d) Compute the limit :

$$\lim_{h\to 0}\frac{\sqrt[3]{27+h}-3}{h}$$

2. Attempt **any 4** parts. (5×4=20)

(a) If $f(x)=x^{1/3}$, show that

$$\frac{f(x+h)-f(x)}{h}=\frac{1}{(x+h)^{2/3}+(x+h)^{1/3}.x^{1/3}+x^{2/3}}$$

Hence find $f'(x)$.

(b) Define Continuous function.

Given functions f and g are discontinuous at a point x=a. Is f+g necessarily discontinuous at x=a ? If not, give example.

(c) State Intermediate Value Theorem.

Using this prove that the equation $x^7 - 5x^5 + x^3 - 1 = 0$ has atleast one solution between $-1$ and 1

(d) If $f(x) = \dfrac{x-1}{2x^2 - 7x + 5}$ when $x \neq 1$,

$f(x) = -\dfrac{1}{3}$, when $x = 1$

Find $f'(1)$.

Give the example of a function which is not differentiable at two points.

(e) Find the maximum of $f(x) = x^2 e^{-x}$ in $[0,4]$.

3 Attempt **any 3** parts : (5×3=15)

(a) What are the conditions required for a System of m Simultaneous equations in n variables to have a solution. When is the solution unique ? When does the matrix equation $\begin{bmatrix} 2 & 4 \\ 5 & \alpha \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \beta \\ 10 \end{bmatrix}$ possess no solution.

(b) Find the equation of the plane passing through the points (1, 2, 3), (2, 3, 4) and (3, 5, 6). Where does the plane cut the axes.

(c) Define Linear Independence of vectors for a set of vectors $S = \{x_1, x_2, \ldots, x_n\}$. If a set of vectors $A = \{a, b, c\}$ is linearly independent, check whether the set $B = \{a-b, b+2c, c\}$ is also linearly independent.

(d) Prove the triangular inequality

$$\|a + b\| \leq \|a\| + \|b\|$$

4. Attempt **any 5** parts : (5×5=25)

(a) A firm has two factories producing identical good X.

The cost functions for the factories are $C_1 = 200 + \frac{1}{100}x_1^2$

and $C_2 = 200 + x_2 + \frac{1}{300}x_2^2$. How much should the good be produced in each factory so as to meet the total order of 2000 units of X.

(b) A consumer has the following utility function $u = 3x^2y^3$. Find his demand for x and y if $p_x = 2$, $p_y = 3$ and total income = 15. Check whether the second order conditions of utility maximisation are satisfied.

(c) Define Homogeneous and Homothetic functions of 2 variables. Show that the function $f(x,y) = xy^2 + x^3$ is homogeneous of degree 3. Is the function $f(x,y) = a \ln x + b \ln y$ homogeneous and homothetic ?

(d) A producer has the following production function :

$X = L^{1/2} + K^{1/2}$, where X, L and K denotes output, labour and capital respectively. Price of labour is Rs. 4 per unit and price of capital is Rs. 5 per unit respectively. Determine the values of L and K for producing 45 units of output at minimum cost. Verify the second order conditions.

(e) State Extreme Value Theorem.

Let $f(x,y) = 4x - 2x^2 - 2y^2$, $S = \{(x,y) : x^2 + y^2 \leq 25\}$

Compute $f_1'(x,y)$ and $f_2'(x,y)$ then find the Stationary point for f.

P.T.O.

(f) Define Concave and Convex Functions.

Let $f(x,y) = x + y - e^x - e^{x+y}$. Is f concave/convex.