Calculus – I

M.C.Q’s

Q1. Find the vertical asymptote(s)(if any) of the graph of the function $3\times(\times^{2}-9)$.

a. $\times=3$
b. $\times=0$
c. None
d. $\times=3,-3$

Q2. Which of these is the polar description of the curve $\times^{2}+ y^{2} =2c\times$? here C is the constant.

a. $r=c\mathrm{Cos}\theta$
b. $r=2c\mathrm{Cos}\theta$
c. $r^{2}=2c\mathrm{Cos}\theta$
d. $r^{2}=c\mathrm{Cos}\theta$
e. $r=c\mathrm{Sin}\theta$

Q3. Find the length of the arc from $\theta=0$ to $\theta=2\pi$ of the cardiod $r=1-\mathrm{Cos}\theta$

a. 4 units
b. 8 units
c. 16 units
d. 2 units

Q4. Describe the curve with equation $16x^{2} +16y^{2} -8x +32y=127$

a. Hyperbola with a=b=3 and centre (1/16,-1)
b. Hyperbola with a=b=3 and centre(1/4,-1)
c. Circle with the centre(1/4,-1) and radius 3
d. Circle with the centre(1/16,-1) and radius 3

Q5. Find the coordinates of the points at which the given parametric curve has a horizontal and/or a vertical tangent $x=t^{2}-t+2$ and $y=t^{3}-3t$.

a. Horizontal tangent (2,-2),(4,2) and vertical tangents none
b. Horizontal tangent (2,-2),(2,4) and vertical tangents (-1.375,1.25)
c. Horizontal tangent none and vertical tangents (1.75,-1.375)
d. Horizontal tangent (2,-2),(4,2) and vertical tangents (1.75,-1.375)

Q6. What is the equation for the volume enclosed by revolving f(x) around the x axis between x=a and y=b?

a. ^{2}dx
b. ^{2}dx
c. dx
d. ^{2}dx

Q7. What is the equation for the volume enclosed by revolving the area between f(x) and g(x) (where f(x)<g(x)) around the x axis between x=a and x=b?

a.
b. dx
c. dx
d. dx

Q8.

a. 0
b. ½
c. 1
d. None

Q9.

a. 0
b. 1/e
c. 1
d. e

Q10 if u and v are two functions of x having derivatives of n^{th} order, then

$$(uv)_n = {}^nC_0\, u_n v + {}^nC_1 u_{n-1} v_1 + \dots {}^nC_r u_{n-r} v_r + \dots + {}^nC_n u v_n$$

This theorem is known as

a. Cauchy's theorem
b. Lagrange's theorem
c. Leibnitz's theorem
d. Lipchitz's theorem

Subjective questions:

Q1. Find absolute maximum and minimum values of

$$f(x) = (x-2)^3 + 1$$

soln. : $f'(x) = 3(x-2)^2$

Finding critical values. The derivative exists for all real no. Thus we solve $f'(x) = 0$;

$$3(x-2)^2 = 0$$

$$(x-2)^2 = 0$$

$$x-2 = 0$$

$$x = 2$$

Since there is only one critical value, there are no end pointsb, we can try to apple maximum-minimum principle-2 using second derivative:

$$f''(x) = 6(x-2)$$

we have,

$$f''(2) = 6(2-2) = 0$$

so the maximum- minimum principle 2 does not apply. We cannot use maximum-minimum principle -1 because there are no end points. But note that $f'(x) = 3(x-2)^2$ this is never negative, thus f(x) is increasing everywhere except at x=2 so there is no maximum no minimum. For $x<2$, say x=1, we have $f''(1) = -6 < 0$. For $x>2$, say x=3 we have $f''(3) = 6 > 0$. Thus at x **B**= 2 the function has a point of inflection.

Q2. Business Maximising Revenue. A stereo manufacturer determines that in order to sell x units of new stereo, the price per unit, in dollars, must be $p(x) = 1000 - x$

The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$.

a) Find total revenue R(x).
b) Find total profit P(x).
c) How many units must the company produce and sell in order to maximise profit?
d) What is the maximum profit?
e) What price per unit must be charged in order to make this maximum profit?

Soln. :

a) R(x) = total revenue
= (Number of units).(Price per unit)
= x.p
$=x(1000-x) = 1000x - x^2$

b) P(x) = total revenue – total cost
= R(x) – C(x)
$= (1000x - x^2) - (3000 + 20x)$
$= -x^2 + 980x - 3000$

c) To find the maximum value of P(x), we first find P'(x):
$P'(x) = -2x + 980$;

This is defined for all real no., so the only critical values will come from solving $P'(x) = 0$

$$P'(x) = -2x + 980 = 0$$
$$-2x = -980$$
$$x = 490$$

There is only one critical value. We can therefore try to use second derivative to determine whether we have an absolute max. note that *p"(x) = -2,* a constant.

Thus P"(490) is negative and so profit is maximised when 490 units are produced and sold.

d) The maximum profit is given by 490 units that is
$P(490) = -(490)^2 + 980.490 - 3000$
$= \$ 237,100$
Thus the stereo manufacturer makes a maximum profit of $327,100 by producing and selling 490 stereos.

e) The price per unit needed to make the maximum profit is p= 1000- 490 = $510

Q3. Business minimising inventory costs: A retail appliance store sell 2500 television sets per year. It costs $10 to store one set for a year. To reorder there is a fixed cost $20, plus a fee of $9 per set. How many per year should the store reorder and is lot size, to minimize inventory cost ?

Soln:let x = the lot size, inventory cost is given by

C(x) = (yearly carrying cost.) + (yearly reorder cost)

We consider each component of inventory costs separately

a) Yearly carrying costs. The average amount held in stock is x/2 and it costs $10 per set for storage. Thus,

Yearly carrying cists = (yearly cost per item)(average no. of items)

= 10.

b) yearly reorder costs. We know that xis the lot size and we let N be the no. of reorders each year. Then Nx = 2500. N=2500/x. thus,
yearly reorder cost = (cost of each order)(No. of orders)
= (20+9x)(2500/x)

c) Thus we have
C(x) = 10. + (20 + 9x)
= 5x + + 22500

d) To find a minimum value of C over [1,2500], we must find C'(x);
$C'(x) = 5 - {}^2$

e) C'(x) exists for all x in [1,2500], so the only critical values are those x-values such that C'(x) = 0. We solve C'(x) = 0;

$$5 - 50000/x^2 = 0$$

$$5 = 50,000/x^2$$

$$5x^2 = 50,000$$

$$X^2 = 10,000$$

$$x = \pm 100$$

Since there is only one critical value in [1,2500], that is x=100, we can use the second derivative to see whether it yields a max. or a min. : $C''(x) = 100{,}000/x^3$

C"(x) is positive for all x in [1,2500], so we have a minimum at x = 100. Thus to minimize inventory costs, the store should order 2500/100 or 25 times per year. The lot size should be 100.

Q4. If = ^{n} then show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y = 0$$

soln : $\cos^{-1}() = \log()^n$

$$y = b\cos[n(\log x - \log n)]$$

$$y_1 = = -b\sin[n(\log x - \log n)]\, n.$$

$$xy_1 = -nb\sin[n(\log x - \log n)]$$

$$xy_2 + 1.y_1 = -nb\cos[n(\log x - \log n)]\, n\,.$$

$$= -.\ Y$$

$$x^2y_2 + xy_1 + n^2y = 0$$

Diff. it n-times by Liebnitz theorem

$$x^2y_{n+2} + {}^nC_1(2x)y_{n+1} + {}^nC_2(2)\,y_n + xy_{n+1} + {}^nC_1(1).y_n + n^2y_n = 0$$

$$x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y = 0$$

Q5. Find the volume of the solid generated when the region enclosed by the curve y = , y = 6, y = 0 is revolved about x axis

Soln: the given curves are y = ,

$$Y = 6$$

They intersect at A(4,2)

The volume of solid generated by the shaded portion

= +

= +dx

= +

= -

= 8 + = .

Q6. Find the arc length of the parametric curve $x = e^t \sin t$,

$y = e^t \cos t$ for 0

soln. here $x = e^t \sin t$; $y = e^t \cos t$

$$= e^t[\sin t + \cos t];\ = e^t(\cos t - \sin t)$$

Therefore, $+= e^{2t}[(\sin t + \cos t) + (\cos t - \sin t)^2]$

$$= 2.e^{2t}$$

Hence, the reqd arc length of the curve

=

= =

Q7. Find the equation for a hyperbola passing through the origin with asymptotes y = 2x+1 and y=-2x+3

Soln : Joint eqn of the asymptotes is (y-2x-1)(y+2x-3) = 0

We know that eq. of hyperbola and the joint eqn. of asymptotes differ by a constant.

Therefore let eq. of hyperbola be

$$(y-2x-1)(y+2x-3)+k = 0$$

It passes through (0,0)

$$(-1)(-3) +k = 0$$
$$k = -3$$

$\Rightarrow$ $y^2+2xy-3y-2xy-4x^2+6x-y-2x+3-3 = 0$

$\Rightarrow y^2-4x^2+4x-4y = 0$

$\Rightarrow (y^2-4y)-4(x^2-x) = 0$

$\Rightarrow$ $(y-2)^2-4-4$ – = 1

Q8. Find the tangential and normal components of acceleration of an object that moves along the parabolic path $y = 4x^2$ at the instance the speed is

Soln. the parabola is $y = 4x^2$

$\Rightarrow$ $x=t$, $y=4t^2$

$\Rightarrow$ x= 1, y = 8.1

X = 0, y = 8

Now, v =

Given: v =

$\Rightarrow$ $t^2 =$ = 399/64

Again, p = =

Now for tangential component of acc.

=

= 2/5

Q9. The velocity of a particle moving in space is

(t) = $e^t + t^2$

Find the position vector of a particle if the position at time t = 0 is

+

Soln :(t) = $e^t + t^2$

(t) = Integrating both sided with respect to t, we get

= e^t + + C

At t= 0 +

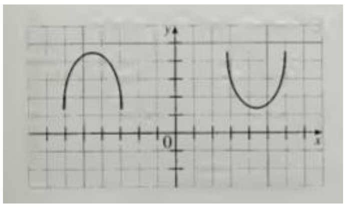
+ =

C = +

The position vector at t is

= e^t + + +

Q10. Find where the graph of $f(x) = x^3 + 3x + 1$ is concave up and where it is concave down



Soln :we find tat $f'(x) = 3x^2 + 3$ and $f''(x) = 6x$. therefore $f''(x) < 0$ if $x < 0$ and $f''(x)>0$ if $x > 0$, so the graph of f is concave down for $x < 0$ and concave up for $x > 0$. The graph is concave down to the left of $x = 0$ and concave up to right. Hence this is the graph of f(x)

Q11 find the curve and radius of curvature for a curve

$$X = e^t\cos t\ ,\ y = e^t\sin t,\ \text{at } t = 0$$

Sol^{n}here $x = e^t \cos t$, $y = e^t \sin t$, z= t

= x

+ and =0. + 2

Now curvature is given by

K = / = =

And radius of curvature

Q12. An efficiency study of the morning shift at a factory indicates that the number of units produced by an average worker 1 hours after 8:00 A.M. may be modeled by the formula 01) = - + 912 + 12t. At what time in the morning is the worker performing most efficiently?

Soln : We assume that the morning shift runs from 8:00 A.M. until noon and that worker efficiency is maximized when the rate of production

$$R(1) = Q'(1) = -3r2 + 181 + 12$$

is as large as possible for 0 <1 < 4. The derivative of R is

$$R'(t)\ Q''(1) = -61 + 18$$

which is zero when t = 3; this is the critical number. Using the optimization criterion. We know that the extrema of R(1) on the closed interval (0, 4] must occur at either the interior critical number 3 or at one (or both) of the endpoints (which are 0 and 4). We find that

$$R(0) = 12\ R(3) = 39\ R(4) = 36$$

so the rate of production R(I) is greatest and the worker is performing most efficiently when i = 3; that is, at 11:00 A.M. The graphs of the production function and its derivative, the rate-of-production function R, are shown in Figure 4.27. Notice that the production curve is steepest and the rate of production is greatest when t = 3.

Q13. A manufacturer estimates that when x units of a particular commodity are produced each month the total cost in dollars will be $C(x) = 1/8x^2 + 4x + 200$ and all units can be sold at a price $P(x) = 49 – x$ \$/unit. Determine the price that corresponds to the maximum profit.

Soln here $C(x) = x^2/8 + 4x + 200$

And $P(x)= 49-x$

Therefore profit function is given by

$F(x) = (49-x)x – (x^2/8 + 4x+200)$

$= -9/8x^2 + 45x -200$

$\Rightarrow$ $f'(x) = -9/4x + 45$ and $f''(x) = -9/4$

For max. or min. $f'(x) = 0 = -9/4x + 45$

X= 20

For this value $f''(x) < 0$

$\Rightarrow$profit f(x) is maximum when x = 20

Therefore, price of a unit = 49-20=\$29

Q14. Find the equation of the hyperbola with vertices (2,4) and (10,4) and foci 10.

Soln. Here vertices of the hyperbola are A' (2,4) and A (10,4)

Now A and A' lie on the line y= 4

Also, center C is the midpoint of A and A'

⇨ C has co-ordinates)= (6,4)

Also, AA' is 2a = 10-2=8, So a=4

And by hypothesis 2ae = 10 $\Rightarrow$ ae = 5 $\Rightarrow$ 4e = 5

Therefore, e= 4/5

Now,

$\Rightarrow$ Equation of the reqd hyperbola is

Q15. Show that the equation for a parabola with axis y = 0 and passing through (3,2) and (2,-3) is .

Soln. To find the equations of a parabola whose axis is y = 0

⇒ Its vertex lies on y = 0

Let be its vertex and be the length of its LR

Therefore, its equation is

==>

Now the parabola is to pass through (3,2) and (2,-3)

Therefore,

⇒

Hence, the reqd equation of the parabola is

Q16. Find the area of the surface generated by revolving the curve , , about the x- axis.

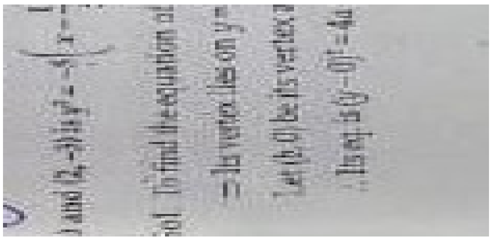
Soln. Here, the given curve is

Its graph is a semicircle with radius, r= 2 units

Shaded region is to revolved about x- axis

Its surface area,

Now,

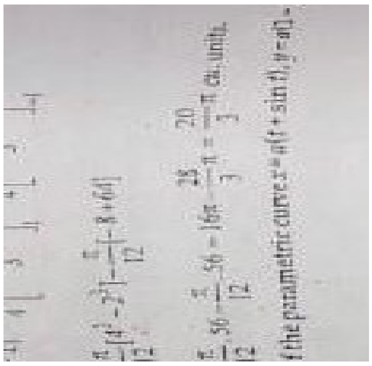


Q17. Find the arc length of the parametric curve .

Soln. The given curve is

Its graph is:

Diff. w.r.t. x



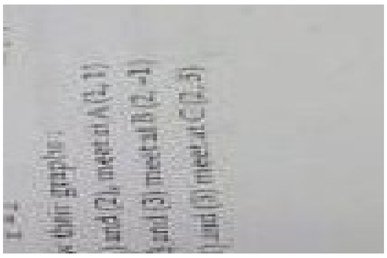
Length of cycloid:

Q18. Find the volume of the solid generated when the region is enclosed by the curve $x=y^2$ and $x=y+2$ is revolved about y-axis.

Soln here the given curves are

$Y^2=x$ ... (i)

$Y+2=x$ ... (ii)



We draw them on a graph

Solving these equations we get $y^2 - y - 2 = 0 \Rightarrow y = 2,-3$

From (i) and (ii) intersect at A(1,-3) and B(4,2)

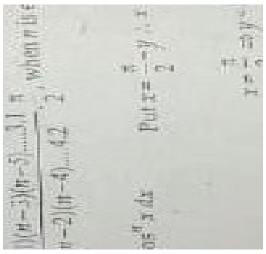
Shaded region is to be revolved about y-axis

The reqd. volume =

= 1/3 .72

Q19. Find the volume of solid generated when the region enclosed by the curves $y=2x-1$, $y=-2x$ and $x = 2$ is revolved about y-axis.

Soln the given curves are



$x = 2$ ... (i)

$y=-2x$ ... (ii)

$y=2x-1$ ... (iii)

now (i) and (ii) meet at A(1,1)

(ii) and (iii) meet at (2,-1)

(i)And(iii) meet at (2,3)

The shaded region is enclosed between these lines and revolved around y-axis

The reqd. volume is

=

=

4 = 20/3.

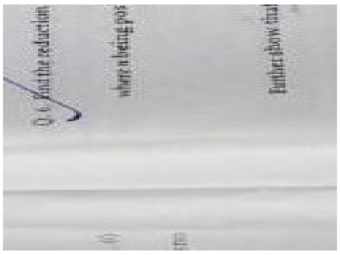
Q20. Sketch the graphn of the curve in polar co-ordinates : r+5 = 5sin

Soln. here the given

curve isr +

5=5sin

r = -5(1-sin



the sin is a line of sym. Of (i) as -1the given curve has no

asymptote.Now tan

Ar ⇒ the curve passes through the pole and

tan = 0,is a tangent at the pole

Now we prepare a table between to view the sym about the line.

			0			
r	-10		-5		0	-5

We draw the graph, this is a cardiode about the line