1. (a) Define fixed point of a function and construct an algorithm to implement the fixed point iteration scheme to find a fixed point of a function. Find the fixed point of $f(x) = 2x(1 - x)$. (6)

(b) Perform four iterations of Newton's Raphson method to find the positive square root of 18. Take initial approximation $x_0=4$. (6)

(c) Find the root of the equation $x^3 - 2x - 6 = 0$ in the interval (2, 3) by the method of false position. Perform three iterations. (6)

2. (a) Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula. (6.5)

(b) Find a root of the equation $x^3 - 4x - 8 = 0$ in the interval (2, 3) using the Bisection method till fourth iteration. (6.5)

(c) Perform three iterations of secant method to determine the location of the approximate root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. (6.5)

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that LU = PA

$$\text{where } A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{pmatrix}$$ (6.5)

(b) Set up the SOR method with w=0.7 to solve the system of equations:

$$3x_1 - x_2 + x_3 = 4$$

$$2x_1 - 6x_2 + 3x_3 = -13$$

$$-9x_1 + 7x_2 - 20x_3 = 7$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Jacobi iteration scheme to solve the system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38$$

Take the initial approximation as $X^{(0)} = (1, 1, 0)$ and do three iterations. (6.5)

4. (a) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data:

x	1	2	4	8
f(x)	3	7	21	73

(6)

(b) Calculate the Newton second order divided difference $\frac{1}{x^2}$ of based on the points x_0, x_1, x_2. (6)

(c) Obtain the Lagrange form of the interpolating polynomial for the following data:

x	1	2	5
f(x)	-11	-23	1

(6)

5. (a) Find the highest degree of the polynomial for which the second order backward difference approximation for the first derivative

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0-h) + f(x_0-2h)}{2h}$$

provides the exact value of the derivative irrespective of h. (6)

(b) Derive second-order forward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{-3f(x_0)+4f(x_0+h)-f(x_0+2h)}{2h}$$

(6)

(c) Approximate the derivative of f(x) = Sin x at $x_0 = \pi$ using the second order central difference formula taking h = ½, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the Simpson's rule, approximate the value of the integral $\int_2^5 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Apply Euler's method to approximate the solution of initial value problem $\frac{dx}{dt} = \frac{e^t}{x}$, $0 \le t \le 2$, $x(0) = 1$ and N = 4.

Given that the exact solution is $x(t) = \sqrt{2e^t - 1}$, compute the absolute error at each step. (6.5)

(c) Apply the optimal RK2 method to approximate the solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \leq t \leq 2$, $x(1) = 1$ taking the step size as $h = 0.5$. (6.5)