[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1049 **C**

Unique Paper Code : 32351502

Name of the Paper : BMATH512: Group Theory-II

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Question No. **1** has been divided into **10** parts and each part is of **1.5** marks.
4. Each question from Q. Nos. **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each Question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) A group of order p^2, p is a prime is always isomorphic to Z_{p2}.

P.T.O.

(ii) A group of order 15 is always cyclic.

(iii) A group of order 14 is simple.

(iv) The smallest positive integer n such that there are two non-isomorphic groups of order n is 6.

(v) Every inner automorphism induced by an element 'a' of group G is an automorphism of G.

(vi) A abelian group of order 12 must have an element of order either 2 or 3.

(vii) U(105) is isomorphic to external direct product of U(21) and U(5).

(viii) Center of a group G is always a subgroup of normalizer of A in G, where A is any subset of G.

(ix) $Aut(Z_{10})$ is a cyclic group of automorphisms of G.

(x) The largest possible order for an element of $Z_{20} \oplus Z_{30}$ is 60.

2. (a) Define inner automorphism induced by an element 'a' of group G and find the group of all inner automorphisms of D_4.

(b) Defile the characteristic and the commutator subgroup of a group. Prove that the centre of a group is characteristic subgroup of the group.

(c) Let G′ be the subgroup of commutators of a group G. Prove that G/G′ is abelian. Also, prove that if G/N is abelian, then $N \geq G'$.

3. (a) Determine the number of cyclic subgroups of order 15 in $Z_{90} \oplus Z_{36}$.

(b) Define the internal direct product of the subgroups H and K of a group G. Prove that every group of order p^2, where p is a prime, is isomorphic to Zp^2 or $Zp \oplus Zp$ (external direct product of Zp with itself).

(c) Consider the group G = {1, 9, 16, 22, 29, 53, 74, 79, 81} under multiplication modulo 91. Determine the isomorphism class of G.

4. (a) Show that the additive group Z acts on itself by $z.a = z+a$ for all $z, a \in Z$.

(b) Show that an action is faithful if and only if its kernel is the identity subgroup.

(c) Let G be a group. Let H be a subgroup of G. Let G act by left multiplication on the set A of all left cosets of H in G. Let π_H be the permutation representation of G associated with this action. Prove tha,t

(i) G acts transitively on A

(ii) The stabilizer of the point $1H \in A$ is the subgroup H.

(iii) $\text{Ker } \pi_H = \cap_{x \in G} xHx^{-1}$

5. (a) Let G be a permutation group on a set A (G is subgroup of S_A), let $\sigma \in G$ and let $a \in A$. Prove that $\sigma G_a \sigma^{-1} = G_{\sigma(a)}$, here G_x denotes stabilizer of x. Deduce that if G acts transitively on A then $\cap_{x \in G} \sigma G_a \sigma^{-1} = 1$.

(b) Show that every group of order 56 has a proper nontrivial normal subgroup.

(c) State Index theorem and prove that a group of order 80 is not simple.

6. (a) State the Class Equation for a finite group G. and use it to prove that p-groups have non trivial centers.

(b) Prove that group of order 255 is always cyclic.

(c) Show that the alternating group ~~As~~ A5 does not contain a subgroup of order 30, 20, or 15.