[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1231 **C**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) Scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the given **eight** questions are compulsory to attempt.

3. Do any **two** parts from each of the given **eight** questions.

4. Marks for each part are indicated on the right in brackets.

P.T.O.

SECTION I

1. (a) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as: For $m, n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \leq N_0$: $n = km$. Then show that $\leq$ is an order relation on N_0. (2½)

(b) If '1', '2', '3' denote chains of one, two, three elements respectively and $\overline{3}$ denotes anti chain of three elements, then draw the Hasse diagram for the dual of $L \oplus K$ when $L = \overline{3}$ and $K = 1 \oplus (2 \times 2)$. (2½)

(c) Define maximum and a maximal element of a partially ordered set P. Give an example each for both definitions. (2½)

2. (a) Let P and Q be finite ordered sets and let $\psi: P \to Q$ be a bijective map. Then show that the following are equivalent :

(i) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

(ii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q (3)

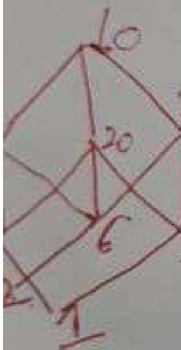
(b) Define upper bound and lower bound of a subset S of a partially ordered set P. Construct an example of a partially ordered set P and its subset S and give the set of all upper bounds and lower bounds of S. (3)

(c) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi: P \to Q$ and $\psi: Q \to P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \to S$ denotes the identity map on S given by: $id_S(x) = x$, $\forall\, x \in S$. (3)

SECTION II

3. (a) Let $D_{60} = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\leq$' is defined on D_{60} by $m \leq n$ if and only if m divides n then show that D_{60} does not form a lattice. Also Draw the diagram of D_{60} and find elements a, b, c, d $\in D_{60}$ such that $a \vee b$ and $c \wedge d$ do not exist in D_{60}. (5½)



$D_{60} - 2\frac{1}{2}$ (20, 30)

(b) Define sublattice of a lattice. Prove that every chain of a lattice L is a lattice and also a sublattice of L. (5½)

(c) Define modular lattice. Prove that a homomorphic image of modular lattice is modular. (5½)

4. (a) Let L be a lattice. For any a, b, c $\in$ L, show that the following inequalities hold :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \geq c \Rightarrow a \wedge (b \vee c) \geq (a \wedge b) \vee c$

(5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define

$$a \leq b :\Leftrightarrow a \vee b = b$$

then show that $(L, \leq)$ is a lattice ordered set.

(5)

(c) Let L_1 and L_2 be distributive lattices. Prove that the product $L_1 \times L_2$ is a distributive lattice.

(5)

SECTION III

5. (a) A voting-machine for three voters has YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the corresponding contact diagram and the switching/circuit diagram.

(5½)

11
111

$p = x_1x_2x_3' + x_1x_2'x_3 + x_1'x_2x_3 + x_1x_2x_3$ 2

$-x_1-x_2-x_3'$

2

b_1
b_2 $-p-$ $1\frac{1}{2}$
b_3

$x_1 - x_2 - x_3$

(b) Show that a Boolean Algebra is relatively complemented. (5½)

$y = (a \vee x') \wedge b$ 2

(c) Simplify the polynomial :

$$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5½)

$x'y + x'z + y'z + xy'$

As $x'y + xy' + y'z$

6. (a) Define a system of normal forms. Find conjunctive normal form for $p = y'z' + x'yz$. (5)

000 100 011 1

$(x'+y+z')(x+y+z)(x'+y+z)(x'+y'+z)(x'+y+z')$

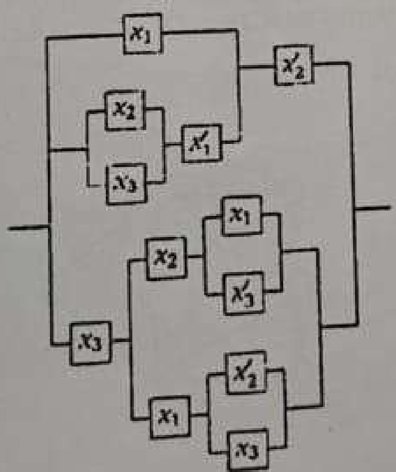
(b) Simplify the Boolean expression :

$$f = w'xy'z + w'xyz + w'xyz' + wxy'z + wxyz + wxyz' + wx'y'z + wx'yz$$

$wz + xy + w'xz$

using Karnaugh Diagram. (5)

(c) Find the symbolic gate representation of the contact diagram : (5)



P.T.O.

$(x_1 + (x_2 + x_3)x_1')x_2' + x_3(x_2(x_1 + x_3') + x_1(x_2' + x_3))$

SECTION IV

7. (a) (i) Show that the sum of the degrees of the vertices of a pseudograph is an even number equal to twice the number of edges. 3

(ii) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have?

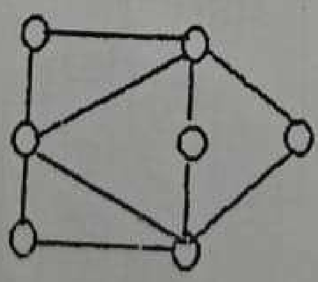
|E|=12 $2\frac{1}{2}$ (5½)

(b) (i) Define the degree sequence of a graph. Does there exist a graph with following degree sequence 6, 6, 5, 5, 4, 4, 4, 4, 3? 1 4 $1\frac{1}{2}$

(ii) Show that the number of vertices of odd in a graph must be even. 3 (5½)

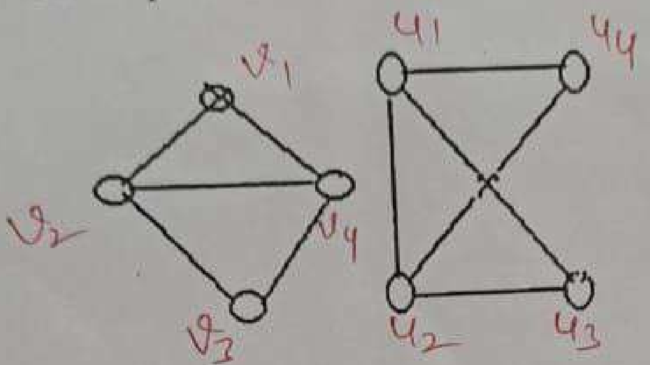
(c) (i) What is a bipartite graph? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite. 1

$1\frac{1}{2}$



(ii) Define isomorphism of graphs. Also label the following graphs so as to show an isomorphism. (5½)

1½

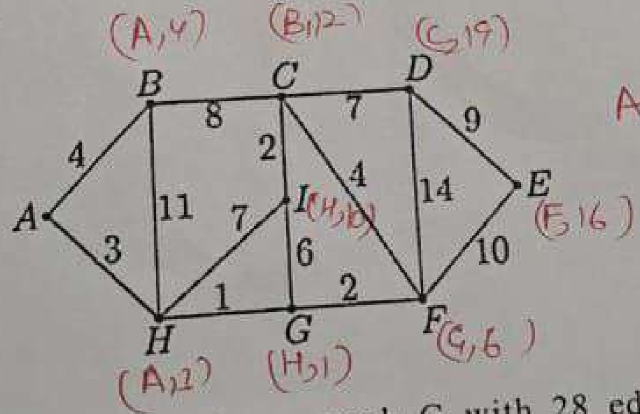


V1 – U3
V2 – U1
V3 – U4
V4 – U2
1½

8. (a) Construct a Gray Code of length 3 using the concept of Hamiltonian Cycles. 3 (5½)

2½ -marks

(b) Apply Dijkstra's algorithm to find a shortest path from A to all other vertices in the weighted graph shown. (5½)



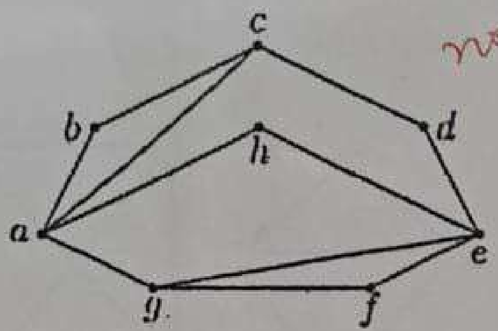
All short Path.

1½ Marks For Path

(c) (i) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 6?

1

(ii) Define Eulerian circuit. Is the given graph Eulerian? Give reasons for your answer.



not eulerian

$1\frac{1}{2}$

(5½)