Complex Analysis

Ques-1) Show that the limit of function $f(z) = \left(\frac{z}{\bar{z}}\right)$ does not exist as $z \to 0$.

Ques-2)Using $\epsilon - \delta$ definition, prove that $\lim_{z \to z_0} Re(z) = Re(z_0)$.

Ques-3) If both sum and product of two complex numbers are real then show that either the numbers are real or one is complex conjugate of other.

Ques-4) Prove that if a set contains each of its accumulation points, then it must be a closed set

Ques-5) Using modulus properties, find upper and lower bound for $|z^4 - 3z + 1|^{-1}$ whenever $|z| = 2$.

Ques-6)Sketch the set of points determined by $|z + i| \leq 3$.

Ques-7)What is the largest domain in which function $w = f(z) = z^2$ is one-one.

Ques-8) Show that the limit of function $f(z) = \left(\frac{Re(z^2)}{|z|^2}\right)$ does not exist as $z \to 0$.

Ques-9) Find $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$.

Ques-10)Find and skech, showing corresponding orientations, the images of hyperbolas

$$x^2 - y^2 = c_1 \ (c_1 < 0) \text{ and } 2xy = c_2 (c_2 < 0)$$

Ques-11)Sketch the region onto which the sector $r \leq 1, 0 \leq \theta \leq \pi/4$ is mapped by the transformation $w = z^4$.

Ques-12)Evaluate $\lim_{z \to \infty} \frac{z^2+1}{z-1} = \infty$.

Ques-13)Show that $\lim_{z \to z_0} f(z)g(z) = 0$ if $\lim_{z \to z_0} f(z) = 0$ and if there exists a positive number M such that $g(z) \leq M$ for all z in neighbourhood of z_0.

Ques-14) Let a function f be analytic everywhere in a domain D. Prove that if $f(z)$ is real valued for all $z \in D$, then $f(z)$ must be constant throughout D.

Ques-15)If $f'(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.

Ques-16)Sketch the set of points determined by $|z - 2i| \geq 2$.

Ques-17)Discuss the image of closed region $x \geq 0,\ y \geq 0,\ xy \leq 1$ under the map $w = z^2$.

Ques-18) Determine the singular points of the function

$$f(z) = \frac{z^3 + 1}{z^2 - 3z + 2}$$

Ques-19) Show that the function $f(z) = 2xy + i(x^2 + y^2)$ is nowhere analytic.

Ques-20) Suupose that $f(z_0) = g(z_0) = 0$ and that that $f'(z_0)$ and $g'(z_0)$ exist, where $g'(z_0) \neq 0$, then show that

$$\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{f'(z_0)}{g'(z_0)}$$

Ques-21) Determine where $f'(z)$ exists and find its value when $f(z) = x^2 + iy^2$.

Ques-22) Show that for $f(z) = z - \overline{z}$, $f'(z)$ does not exist at any point.

Ques-23) Show that $f'(z)$ does not exist for $f(z) = e^x e^{-iy}$ at any point.

Ques-24) Find $f''(z)$ when $f(z) = iz + 2$.

Ques-25) Determine where $f'(z)$ exists and find its value when $f(z) = zImz$.

Ques-26)Find all values of z such that $e^z = -2$.

Ques-27) State why the function $f(z) = 2z^2 - 3 - ze^z + e^{-z}$ is entire.

Ques-28) Use cauchy-riemann equations to show that $f(z) = \exp \overline{z}$ is not analytic anywhere.

Ques-29)Show that $\overline{\exp(iz)} = \exp(i\overline{z}))$ if and only if $z = n\pi, (n \in \mathbb{Z})$.

Ques-30) Show that if e^z is real, then $Imz = n\pi, \ (n \in \mathbb{Z})$.

Ques-31)Show that $|\exp(z^2)| \leq \exp(|z|^2)$.

Ques-32) Prove that $|\exp(-2z)| < 1$ if and only if $Re\, z > 0$.

Ques-33) Show that $Log(1-i) = \frac{1}{2}ln2 - \frac{\pi}{4}i$.

Ques-34) Evaluate $\int_1^2 (\frac{1}{t} - i)^2 dt$.

Ques-35) Evaluate $\int_C \frac{z+2}{z}$ where C is semicircle $z = 2e^{i\theta}(0 \leq \theta \leq \pi)$.