Probability

Q.1 An experiment consists of rolling a die once. Let X be the outcome. Let F be the event $\{X = 6\}$, and let E be the event $\{X > 4\}$. We assign the distribution function $m(\omega)=1/6$ for $\omega = 1, 2, \ldots, 6$. Thus, $P(F)=1/6$. Now suppose that the die is rolled and we are told that the event E has occurred. This leaves only two possible outcomes: 5 and 6. In the absence of any other information, we would still regard these outcomes to be equally likely, so the probability of F becomes 1/2, making $P(F|E)=1/2$.

Q.2 In the Life Table (see Appendix C), one finds that in a population of 100,000 females, 89.835% can expect to live to age 60, while 57.062% can expect to live to age 80. Given that a woman is 60, what is the probability that she lives to age 80? This is an example of a conditional probability. In this case, the original sample space can be thought of as a set of 100,000 females. The events E and F are the subsets of the sample space consisting of all women who live at least 60 years, and at least 80 years, respectively. We consider E to be the new sample space, and note that F is a subset of E. Thus, the size of E is 89,835, and the size of F is 57,062. So, the probability in question equals 57,062/89,835 = .6352. Thus, a woman who is 60 has a 63.52% chance of living to age 80.

Q3. Theorem 4.1 If $P(E) > 0$ and $P(F) > 0$, then E and F are independent if and only if $P(E \cap F) = P(E)P(F)$.

Q4. Example 4.8 It is often, but not always, intuitively clear when two events are independent. In Example 4.7, let A be the event "the first toss is a head" and B the event "the two outcomes are the same." Then P(B|A) = P(B ∩ A) P(A) = P{HH} P{HH,HT} = 1/4 1/2 = 1 2 = P(B).

Q5. X be a continuous rv with pdf f (x) and cdf F(x). Then for any number a, $P(X > a) = 1 - F(a)$ and then prove for any two numbers a and b with $a < b$, $P(a \le X \le b) = F(b) - F(a)$.

Q6 Let X denote the amount of space occupied by an article placed in a 1-ft3 packing container. The pdf of X is f (x) = (90x 8 (1 − x) 0 < x < 1 0 otherwise Then what is $P(X \le 0.5)$ and $P(0.25 < X \le 0.5)$?

Q7 If X is a continuous rv with pdf f (x) and cdf F(x), then at every x at which the derivative F 0 (x) exists, F0 (x) = f (x). e.g. for the previous example, we know the cdf for X is F(x) = ? ?? ?? 0 x ≤ 0 10x 9 − 9x 10 0 < x < 1 1 x ≥ 1 Then the derivative of F (x) exists on (−∞, ∞) and we get F 0 (x) = 90x 8 − 90x 9 for 0 < x < 1 and F0 (x) = 0 for −∞ < x ≤ 0 and 1 ≤ x < ∞, which is just the pdf of X

Q8 Suppose MX(t) = 1 8 e −5t + 1 4 e t + 3 8 e 7t . Find a formula for E[Xn].

Q9 Suppose MX(t) = 9 (3−t) 2 . Find a formula for E[Xn]. 3 Suppose MX(t) = e [3e t−3] . Find V ar[X]. 4

Q10Supppose X is a random variable which takes on the values 1, 2, 3, and 4, with probabilities .1, .2, .3, and .4, respectively. Find a formula for MX(t).

Q11 Suppose P[X = n] = q n−3p for n ≥ 3. Find a formula for MX(t).

Q12 Suppose X is a continuous random variable with density function f(x) = ½1 2 x 2 e −x if x ≥ 0 0 otherwise. Find the moment-generating function MX(t).

Q13Find the moment generating function corresponding to

a) f(x) = 1 c , 0 < x < c.

b) f(x) = 2x c 2 , 0 < x < c.

Q14 number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Q15 4 cards are drawn from a well – shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Q16. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine

I)P(2) (ii) P(1 or 3) (iii) P(not 3)

Q17. In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets.

Q18. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that (a) you both enter the same section?

Q19. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Example 15 Find the probability that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all Kings (ii) 3 Kings (iii) atleast 3 Kings.

Q20. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) (ii) The student has opted neither NCC nor NSS.

Q21 A worn machine is known to produce 10% defective components. If the random variable X is the number of defective components produced in a run of 3 components, find the probabilities that X takes the values 0 to 3.

Q22 A worn machine is known to produce 10% defective components. If the random variable X is the number of defective components produced in a run of 4 components, find the probabilities that X takes the values 0 to 4.

Q23 In a box of switches it is known 10% of the switches are faulty. A technician is wiring 30 circuits, each of which needs one switch. What is the probability that (a) all thirty work, (b) at most 2 of the circuits do not work?

Q24 Using the Binomial model, and assuming that a success occurs with probability 1 5 in each trial, find the probability that in 6 trials there are (i) 0 successes (ii) 3 successes (iii) 2 failures.

Q25 A die is thrown repeatedly 36 times in all. Find E(X) and V (X) where X is the number of sixes obtained.

Q26The probability that a mountain-bike rider travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders: (a) exactly one has a burst tyre (b) at most three have a burst tyre (c) two or more have burst tyres.

Q27 An examination consists of 10 multi-choice questions, in each of which a candidate has to deduce which one of five suggested answers is correct. A completely unprepared student may be

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assumed to guess each answer completely randomly. What is the probability that this student gets 8 or more questions correct?

Q28The probability that a machine will produce all bolts in a production run within speci- fication is 0.998. A sample of 8 machines is taken at random. Calculate the probability that (a) all 8, (b) 7 or 8, (c) at least 6 machines will produce all bolts within specification 5. The probability that a machine develops a fault within the first 3 years of use is 0.003.

Q29 If 40 machines are selected at random, calculate the probability that 38 or more will not develop any faults within the first 3 years of use.

Q30. A computer installation has 10 terminals. Independently, the probability that any one terminal will require attention during a week is 0.1. Find the probabilities that (a) 0, (b), 1 (c) 2, (d) 3 or more, terminals will require attention during the next week.

Q31. The quality of electronic chips is checked by examining samples of 5. The frequency distribution of the number of defective chips per sample obtained when 100 samples have been examined is: No. of defecives 0 1 2 3 4 5 number of samples 47 34 16 3 0 0 Calculate the proportion of defective chips in the 500 tested. Assuming that a Binomial distribution holds, use this value to calculate the expected frequencies corresponding to the observed frequencies in the table distribution of X.

Q32A machine is built to make mass-produced items. Each item made by the machine has a probability p of being defective. Given the value of p, the items are independent of each other. Because of the way in which the machines are made, p could take one of several values. In fact p = X/100 where X has a discrete uniform distribution on the interval [0, 5]. The machine is tested by counting the number of items made before a defective is produced. Find the conditional probability distribution of X given that the first defective item is the thirteenth to be made.

Q 33Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(a) Both Anil and Ashima will not qualify the examination.

(b) Atleast one of them will not qualify the examination

Q 34 A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be (i) red, (ii) yellow, (iii) blue, (iv) not blue, (v) either red or blue.

Q35 find probability of atmost 4 heads occur when coin tossed 10 times.

Q36 A committee of two persons is selected from two men and two women. What is the probability that the committee will have (a) no man? (b) one man? (c) two men?