B.A. (Prog) Sem VI

Paper: Numerical Analysis

Q 1. Find a real root of the equation $x^4 - x - 10 = 0$ using bisection method correct to three decimal places.

Q 2. Use Regula-Falsi method to find a real root of equation $x^3 - 4x - 9 = 0$ correct to three decimal places.

Q 3. Find a root of equation $x^3 + x^2 + x + 7 = 0$ by secant method using four iterations.

Q 4. . Find a real root of the equation $3x = \cos x + 1$ using Newton-Raphson method correct to three decimal places.

Q 5. Find a root of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$ correct to three decimal places which lies between 2 and 3, using Newton's method.

Q6. Perform three iterations of the Newton-Raphson method to solve the system of equations $x^2 + xy + y^2 = 7$ and $x^3 + y^3 = 9$ with initial approximation as (1.5, 0.5).

Q 7. Solve the system of equations $3x_1 + 6x_2 + x_3 = 16$ $2x_1 + 4x_2 + 3x_3 = 13$ and $x_1 + 3x_2 + 2x_3 = 9$ by Guass elimination method.

Q 8. Solve the system of equations $x + 2y + z = 8,\ 2x + 3y + 4z = 20,\ 4x + 3y + 2z = 16$ by Guass-Jordon method.

Q 9. Solve the following system of equations by Jacobi method :

$$5x + 2y + z = 12\ ;\ x + 4y + 2z = 15\ ;\ x + 2y + 5z = 20$$

Q 10. . Solve the following system of equations by Gauss-Seidel method :

$10x + y + 2z = 44\ ;\ 2x + 10y + z = 51;\ x + 2y + 10z = 61$ correct to four significant digits.

Q 11. From the following table Interpolate the value of $y(x)$ using Lagrangian polynomial at

(i) 2.8 (ii) 3.1

x :	2.0	3.0	4.0
$y(x)$:	6.6	9.2	8.6

Q 12. Given the table of values as :

x :	2.0	2.25	2.5	2.75	3.0
$y(x)$:	9.00	10.06	11.25	12.56	14.00

Find $y(2.35)$ using Newton's forward difference interpolation formula.

Q13. From the following table , find $f(0.7)$ approximately.

x:	0.1	0.2	0.3	0.4	0.5	0.6

Q14. Using Newton's divided difference formula, evaluate f(8) and f(15) from following data :

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Q15. Given the following values of $f(x)$ = log x, find the approximate value of $f'(2.0)$ a $f''(2.0)$ using the method based on linear and quadratic interpolation. Also obtain an upper bound on the error.

i	0	1	2
x_i	2.0	2.2	2.6
f_i	0.69315	0.78846	0.95551

Q 16. Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

Using (i) trapezoidal rule, and (ii) Simpson's rule. Obtain a bound for the errors.

Q 17. Find the value of the integral

$$I = \int_2^3 \frac{\cos 2x}{1+\sin x}\,dx$$

Using Guass-Legendre two and three point integration rules.

Q 18. Compute

$I_p = \int_0^1 \frac{x^p}{x^3+10} dx$ for p= 0, 1

Using trapezoidal and Simpson's rules with the number of points 3, 5 and 9. Improve the results using Romberg integration.

Q 19. Given $\frac{dy}{dx} = xy$ with $y(1) = 5$.

Find the solution correct to 3 decimal in the interval (1, 1.3] by using modified Euler's method with step size $h = 0.1$.

Q 20. Use Runge-Kutta method of second order to find approximate value of y correct to three decimal places at $x = 1.1$, given $\frac{dy}{dx} = 3x + y^2$ and $y = 1.2$ when $x = 1$.

Q 21. Given $\frac{dy}{dx} = 1 + y^2$ and $y(0) = 0$. Find $y(0.4)$ using Runge-Kutta fourth order method with step size of 0.2.

Q 22. Solve the initial value problem

$$u' = -2tu^2, \qquad u(0) = 1$$

Using the following methods :

(i) Euler method
(ii) Mid-point method

Q 23. Solve the boundary value problem

$$u'' = u + x$$

$$u(0) = 0, \qquad u(1) = 0$$

With $h = \frac{1}{4}$ using the second order method.

Q 24. Prove the relation : $\nabla - \triangle = -\triangle\nabla$.

Q 25. Find the unique polynomial $P(x)$ of degree 2 or less such that

$$P(1) = 1, \qquad P(3) = 27, \qquad P(4) = 64$$

Using the Lagrange interpolation formula and the Newton's divided difference formula. Evaluate $P(1.5)$.

$|x + y + z = 6$

$$3x + 3y + 4z = 20$$

$$2x + y + 3z = 13$$

Using the Gauss elimination method.

Q 27. Apply Newton-Raphson's method to determine a root of the equation

$$f(x) = \cos x - x e^x = 0$$

such that $|f(x^*)| < 10^{-8}$, where x^* is the approximation to the root.

Q 28. Perform five iterations of the bisection method to obtain the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

Q 29. Evaluate the integral

$$I = \int_0^1 \frac{dx}{1+x}$$

Using (i) composite trapezoidal rule, and (ii) composite Simpson's rule, with 2, 4 and 8 equal subintervals.

Q 30. Solve the differential equation $\frac{dy}{dx} = x + y^2$ and $y(0) = 1$ at 0.2 using Runge-Kutta fourth-order method.