ALGEBRA III

Q1. Describe all the subrings of the ring of integers.?

Q2. Prove that intersection of any collection of subrings of a ring R is a subring of R.?

Q3. Prove that the centre of a ring is a subring.?

Q4. Find all the units,zero-divisors,idempotents and nilpotents elements in $Z_3 + Z_6$?

Q5. Suppose that R is a commutative ring without zero-divisors. Show that all the non-zero elements of R have the same additive order.?

Q6. Suppose that R is a commutative ring without zero-divisors. Show that the characteristic of R is zero or prime.?

Q7. Show that any finite field has order p^n where p is prime.?

Q8. Let R be ring with m elements. Show that the characteristic of R divides m.?

Q9. Prove that Zp is a field iff p is a prime.?

Q10. Find all the maximal ideals in –

(i)Z_8 (ii)Z_{10}

Q11. In a ring of integers , find a positive integer a such that –

(i)<a>=<3><4> (ii) <a>=<6><8>

(iii) <a>=<2> + <3> (iv) <a>=<3> + <6>

Q12. Prove that only ideals of a field F are {0} and F itself.?

Q13. Prove that I=<2+2i> is not a prome idealofZ[i].? How many elements are in Z[i]/I.? What is the characteristic of Z[i]/I.?

Q14. Show that $Z_3[x]/<x^2 + x + 1>$ is not a field.?

Q15. Show that $R[x]/,x^2 + 1>$ is a field.?

Q16. Prove that R/A is an integral domain iff A is prime.?

Q17. Prove that R/A is a Field iff A is maximal.?

Q18. Is the ring 2Z isomorphic to the ring 4Z.?

Q19. Determine all the ring homomorphisms from Z6 to Z6.?

Q20. Determine all the ring homomorphisms from Z to Z.?

Q21. Let Ǿ be a ring homomorphism from a commutative ring R onto a commutative ring S and let A be an ideal of S –

a)If Ais prime in S, show that $Ǿ^{-1}(A) = \{x € R \mid Ǿ(x) € A\}$ is a prime in R.

b)If Ais maximal in S, show that $Ǿ^{-1}(A)$ is maximal in r.

Q22. Show that for any positive integer m, the mapping of Ø:Z -> Zm is a ring homomorphism.?

Q.23 Examine the following Sets for Linear Independence

a) {(1,2,3),(2,-1,6),(1,0,2)}

b) {(1,0,0),(2,-1,0),(3,9,1),(0,1,2)}

c) {(5,2,4,3),(10,4,8,6)}

Q.24 Prove that any Linearly independent Set can be extended to form a Basis.

Q.25 Define Basis and Dimension of a Set.Find Dimension of the following Sets:

a) W={(a,b,c,d)|a=2b, c=d}

b) W'={(a,b,c,d)|a=b=c=d}

c) W+W'

d) Intersection of the Sets W and W'

Q.26 Define Linear Transformation .

Which of following are Linear Transformations:

Define T:$R^3 \rightarrow R^3$ such that:

a) T(a,b,c)=(-a,c,b)

b) T(a,b,c)=(a,b,0)

c) T(a,b,c)=(ab,ac,bc)

JUSTIFY

Q.27 Define Matrix of Linear Transformation.Find Matrix of Linear Transformation for each of the following wrt standard Basis.

Define T:$R^2 \to R^2$ such that

a) T(a,b)=(b,a)
b) T(a,b)=(-a,-b)
c) T is defined as reflection of a point about Xaxis
d) T is defined as reflection of a point about Yaxis
e) T rotates the point by an angle 45 degrees in anticlockwise direction.

Q.28 State and Prove Dimension Theorm.

Q.29 Let T:$R^3 \to R^2$ as

T(a,b,c)=(a-b,2c).

Find R(T),N(T) and Dimension of N(T)

Q.30 Let T:$R^2 \to R^2$ as

T(a,b)=(a+b,a).Is T one-one?Is T onto?.Justify your Answ.er