ALGEBRA –III

Ques 1. Describe all ring homomorphism of Z into Z.

Ques 2. Consider the map det of $M_n(R)$ into R where det(A) is the determinant of the matrix A for A Ɛ $M_n(R)$. Is det a ring homomorphism? Justify your answer.

Ques 3. Show that <nZ, +,.> is a ring.

Ques 4. Describe all ring homomorphism of Z into Z × Z.

Ques 5. Describe all ring homomorphism of Z × Z into Z.

Ques 6. Show that an intersection of subfields of a field F is again a subfield of F.

Ques 7. Find all solutions in Z6 of (a) x2 + 2x + 4 = 0, and (b) x2 + 2x + 2 = 0.

Ques 8. Let R be a commutative ring with unity of characteristic 3. Compute and simplify $(a + b)^9$ for a, b Ɛ R .

Ques 9. Show that the characteristic of an integral domain D must be either 0 or a prime p.

Ques 10. An element a of a ring R is idempotent if a2 = a. Show that a division ring contains exactly two idempotent.

Ques 11. Find all solutions of the equation $x^3 - 2x^2 - 3x = 0$ in Z12.

Ques 12. Solve the equation 3x = 2 in Z7.

Ques 13. Show that the characteristic of a subdomain of an integral domain D is equal to the characteristic of D.

Ques 14. Show that an intersection of subrings of a ring R is again a subring of R.

Ques 15. Describe all units in the ring (a) Q (b) Z5.

Ques 16. Let R be a ring, and let a be a fixed element of R. Let Ia = {x Ɛ R | ax =0}. Show that Ia is a subring of R.

Ques 17. Prove that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

Ques 18. Prove that every field F is an integral domain.

Ques 19. Prove that every finite integral domain is a field.

Ques 20. find the characteristic of Z6 × Z15?

Ques 21. Show that the characteristic of an integral domain D must be either 0 or a prime p.

Ques 22. If W1 and W2 are subspaces of a vector space V, then show that their unions a subspace of V if and only if one of the spaces Wi is contained in the other.

Ques 23. Let S = {p(x) ƐP5 | p(1) = 0 and p(3) = 0}, where P5 is the space of all polynomials in the variable x of degree ≤ 5. Show that S is a subspace of P5.

Ques 24. Given that {u, v, w} is linearly independent, check whether {u−v, v−w, w−u} is linearly independent or not.

Ques 25. Find the value of m such that (m, 7, −4) is a linear combination of the
vectors (−2, 2, 1) and (2, 1, −2).

Ques 26. Find three vectors in R3 which are linearly dependent and are such that any two of them are linearly independent.

Ques 27. Are the vectors u1 = (1, 1, 2, 4), u2 = (2, −1, −5, 2), u3 = (1, −1, −4, 0) and
u4 = (2, 1, 1, 6) linearly dependent in R4?

Ques 28. Consider the subspace U = {(x, y, z)| x − y + z = 0} of R3. Find a basis of U. What is dim U?

Ques 29. Show that S = {(0, 1, 1), (1, 0, 1), (1, 1, 0)} is a basis for R3.

Ques 30. Find the coordinates of $x^2 + 2x - 1$ relative to the ordered basis B = {x + 1, $x^2 + x - 1$, $x^2 - x + 1$} of P2.

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Ques 33. Show that S = {1, x, x2} is a basis for P2, the space of all polynomials of degree ≤ 2.

Ques 34. Find the coordinates of (3, 4, 5) Ɛ R3 relative to the ordered basis B = {(1, 0, 1), (1, 1, 0), (0, 1, 1)} of R3.

Ques 35. Prove that if two vectors in a vector space V are linearly dependent, one of them is a scalar multiple of the other.