

## ANALYSIS-III

Q.1 Find the radius of convergence for Power Series whose  $n$ th term is  $n^2 x^n$ .

Q.2 Let  $f_n(x) = \frac{nx}{1+n^2x^2}$

Show that  $f_n \rightarrow 0$  pointwise on  $\mathbb{R}$ . Also check that if  $f_n \rightarrow 0$  uniformly on  $[0,1]$ . Justify.

Q.3 A Bounded function  $f$  on  $[a,b]$  is integrable iff for each  $E > 0$  there exist a partition  $P$  of  $[a,b]$  such that  $U(f,P) - L(f,P) < E$ .

Q.4 Let  $f(x) = x$ ,  $x$  is rational no. and  $f(x) = 0$ ,  $x$  is irrational, Calculate the upper and lower Darboux integrals for  $f$  on  $[0,b]$ . Is  $f$  integrable on  $[0,b]$ ?

Q.5 Prove that every monotonic function  $f$  on  $[a,b]$  is integrable.

Q.6 Prove that every continuous function  $f$  on  $[a,b]$  is integrable.

Q.7 Determine the convergence/divergence of the following improper integral

$$\int_1^{+\infty} \frac{1}{x^2}$$

Q.8 A Bounded function  $f$  on  $[a,b]$  is Riemann integrable iff it is Darboux integrable.

Q.9 Check for Uniform Convergence

$$f_n(x) = \frac{1}{n} \sin(nx + n) \text{ on } \mathbb{R}.$$

Q.10 Let  $f$  be a bounded function on  $[a,b]$ . If  $P$  and  $Q$  are partitions of  $[a,b]$  and set  $P$  is contained in  $Q$ , then

$$L(f,P) \leq L(f,Q) \leq U(f,Q) \leq U(f,P)$$

Q.11 If  $f$  is a bounded function on  $[a,b]$ , and if  $P$  and  $Q$  are partitions of  $[a,b]$ , then

$$L(f,P) \leq U(f,Q)$$

Q.12 If  $f$  is a bounded function on  $[a,b]$  then  $L(f) \leq U(f)$ .

Q.13 Check for Uniform Convergence

$$f_n(x) = \frac{x}{n} \text{ on } [0,1].$$

Q.14 Check for Uniform and Pointwise Convergence

$$f_n(x) = x^n \text{ on } [0,1].$$