PARTIAL DIFFERENTIAL EQUATION – II

Ques 1.
Prove that a family of spheres $x^2 + y^2 + (z^2 - c^2) = r^2$ satisfies the linear partial differential equation of first order $xp - yq = 0$.

Ques 2.
Prove that the family of right circular cones whose axes coincide with the z-axis $x^2 + y^2 = (z^2 - c^2)(\tan \alpha)^2$ satisfies the linear partial differential equation of first order $xp - yq = 0$.

Ques 3. Calculate the general solution of the first order linear partial differential equation $y^2up + u^2xp = -xy^2$.

Ques 4. Calculate the general solution of the first order linear partial differential equation $y^2u_x - xyu_y = x(u - 2y)$.

Ques 5. Find the solution of the equation $x\,u_x + y\,u_y = 2xy$. With Cauchy data on $u=2$ and $y=x^2$.

Ques 6. Solve the Cauchy problem $(y+u)u_x + u_y = (x-y)$ with $u=1+x$ on $y=1$.

Ques 7. Use the separation of variables $u(x, y) = X(x) + Y(y)$ to solve the equation $u_x^2 + u_y^2 = u$.

Ques 8.
Using the method of separation of variables $u(x, y) = X(x).Y(y)$ to solve the following equations
$u_x + u = u_y$.

Ques 9. **(a)** Show that the equation $u_{tt} + \alpha u_t = c^2 u_{xx}$ represents the

damped wave equation of a string where the damping force is directly
proportional to the velocity and α is a constant.

(b) Show that for a restoring force which is directly proportional to the
displacement of a string the resulting equation is

$u_{tt} + \alpha u_t + bu = c^2 u_{xx}$

Where *b* is a constant.

Ques 10. Derive the one – dimensional heat equation, $u_t = ku_{xx}$
where k is a constant and also show that when the heat lost by radioactive exponential decay of the material in the bar is also considered then the one-dimensional heat equation becomes, $u_t = ku_{xx} + e^{-\alpha x}$ where *h* and a are constant.

Ques 11. Write the one – dimensional heat equation that could be used to determine the temperature in a flat circular disk with the flat surface insulated.

Ques 12. Express Laplace's equation using spherical coordinates.

Ques 13. A tight string 2 m long with a = 30 m/s is initially at rest but is given an initial velocity of 300 sin 4 π x from its equilibrium position. Determine the maximum displacement at the x = 1/8 m location of the string.

Ques 14. Solve the initial boundary – value problem

$Z_{tt} = c^2 z_{xx}$, $0 < x < t$, $t > 0$

$Z(x, 0) = x(1-x)$, $z_t(x, 0) = 0$, $0 <= x <= 1$

$Z(0,t) = z(1,t) = 0$, $t > 0$.

Ques 15 Consider the initial boundary – value problems

$Z_{tt}=c^2 z_{xx}$ f(x), 0<x<l, t>0
Z(x, 0) = f(x), z_t(x, 0) = g(x), 0<=x<=l
Z(0, t) =a, z(l, t)= b, t>0 .

Ques 16. A tight string, π m long and fixed at both ends, is given an initial displacement *f (x)* and an initial velocity *g(x)*. Find an expression for z(x, t).

Ques 17. Solve the initial boundary – value problem
$Z_{tt}=c^2 z_{xx}$, 0<x<Π, t>0
Z(x, 0) = 3 Sinx, z_t(x, 0)= 0, 0<=x<=Π
Z(0,t)= z(Π,t)=0, t>0 .

Ques 18. A long copper rod with insulated lateral surface has its left end maintained at a temperature of 00 C and its right end, at x = 2m, maintained at 100^0 C. Determine the temperature as a function of x and t if the initial condition is given by
T(x,0) = 100x, 0<x<1 and 100, for 1<x<2.

Ques 19. Solve the initial boundary – value problem
$Z_{tt}=c^2 z_{xx}$, 0<x<Π, t>0
Z(x, 0) = 0, z_t(x, 0)= 8 $\sin x^2$, 0<=x<=Π
Z(0,t)= z(Π,t)=0, t>0 .

Ques 20. Solve the partial differential equation $u_{xx}+2u_{xy}-3u_{yy}=0$ with the given initial conditions $u(x, 0) = \text{Sin}x$, $u_y(x,0)=x$.

Ques 21. Transform the equation to the form $z_{xx}-z_{yy}+3z_x-2z_y=0$, $v_{rs}= cv$, c=constant by introducing the new variable $v=-ze^{-(ar+bs)}$ where a and b are undetermined coefficients.

Ques 22. Reduce the following equations in the canonical form after classifying given equation $y^2 z_{xx}+x^2 z_{yy} =0$.

Ques 23. Determine the general solution of the following equation

$y^2 z_{xx}+x^2 z_{yy} =0$.

Ques 24. Determine the general solution of the following equation

$z_{xxx} + z_{yyy} =0$.

Ques 25. Consider the equation to reduce this equation to *canonical form*

$$z_{xx} +z_{yy}-2z_{xy}-3z_x-6z_y=9(2x-y).$$

Ques 26. Solve the initial boundary-value problem

$$u_{tt}- 4u_{xx}= 0,\ 0<x<\infty,\ t>0$$
$$u(x, 0) =x^4,\ 0\le x\le \infty,$$
$$u_t(x, 0) =0,\ \ 0\le x\le \infty,$$
$$u(0, t) = 0,\ t\ge 0.$$

Ques 27. Write the two – dimensional heat equation that could be used to determine the temperature in a flat circular disk with the flat surface insulated.

Ques 28. Solve the initial boundary – value problem
$z_{tt}=c^2 z_{xx}$, $0<x<t$, $t>0$
$z(x, 0) = x$, $z_t(x, 0)=0$, $0<=x<=1$
$z(0,t)= z(1,t)=0$, $t>0$.

Ques 29. Find the solution of the equation $x u_x + y^2 u_y=2xy$. With Cauchy data on $u=3$ and $y=x^2$.

Ques 30. Transform the equation to the form
$3z_{xx}+7z_{yy}+2z_x+z_y=0$, $v_{rs}= cv$, c=constant by introducing the new variable $v=-ze^{-(ar+bs)}$ where a and b are undetermined coefficients.

Ques 31. Calculate the general solution of the first order linear partial differential equation $yup + uxp = -xy^2$.

Ques 32. Solve the signal problem governed by the wave equation

$$4u_{tt}- 3u_{xx}= 0,\ 0 < x < \infty,\ t >0$$
$$u(x, 0) = x,\ 0 \le x \le \infty,$$
$$u_t(x, 0) =0,\ 0 \le x \le \infty,$$
$$u(0, t) = 0,\ t \ge 0.$$

Ques 33. Use the separation of variables $u(x, y) = X(x) +Y(y)$ to solve the equation $4u_x^2+5u_y^2=u$.

Ques 34. Consider the equation to reduce this equation to *canonical form*

$$z_{xx} + x^2 z_{yy} =0.$$