ANALYSIS – 2 QUESTION BANK

Question 1- Determine a condition on $|x-1|$ such will assure that: $|x^2 - 1| < 1/2$

Question 2- For each of the following functions on R to R, find points of relative extrema, the intervals on which the function is increasing, and those on which it is decreasing:

(a) $f(x) = x^2 - 3x + 5$

(b) $g(x) = 3x - 9$

Ouestion 3- Let I be an interval and let f : I to R be differentiable on I. Show that if f' is positive on I then f is strictly increasing on I.

Question 4- Let I be an interval and let f : I to R be differentiable on I. Show that if the derivative f' is Never 0 on I, then either $f'(x) > 0$ for all x in I or $f'(x) < 0$ for all x in I.

Question 5- Let f : R to R, let J be a closed interval in R, and let c is in J. If f2 is the restriction of f to J show that if f has a limit at c then f2 has a limit at c. Show by example that it does Not follow that if f2 has a limit at c, then f has a limit at c.

Question 6- Give an example of a function that is equal to its Taylor series expansion about $x = 0$ For $x > 0$, but is not equal to this expansion for $x < 0$.

Question 7- Evaluate the $\lim (x - \sin x)/x$

Question 8- Show that $\lim (1/n) - (1/[1+n]) = 0$.

Question 9- Alternate the terms of the sequences $(1 + 1/n)$ and $(-1/n)$ to obtain the sequence (X_n) given by $(2, -1, 3/2, -1/2, 4/3, -1/3, 5/4, -1/4, \ldots)$

Determine the values of $\limsup(X_n)$ and $\liminf(X_n)$.

Question 10- Show from the definition that the following is not a Cauchy sequence.

(a) $(\ln n)$

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Question 11- Let f(x) := |x| for $-1 < x < 2$. Calculate L (f ; P) and U(f ; P) for the following partitions:

(a) P1 := (- 1,0, 1, 2),

(b) P2 := (-1, -1/2, 0, ½, 1, 3/2, 2)

Question 12- If f is a bounded function on [a, b] such that f(x)= 0 except for x in { c1, c2, ...,cn} in [a, b], show that U(f) = L(f) = 0.

Question 13- Prove that if f and g are each uniformly continuous on R, then the composite function f o g is uniformly continuous on R.

Question 14- For each of the following functions on R to R, find points of relative extrema, the intervals on which the function is increasing, and those on which it is decreasing:

(a) f(x) = x^2 - 3x+ 5
(b) g(x)= 3x- 4x^2

Question 15- Suppose that f : [0, 2] to R is continuous on [0, 2] and differentiable on (0, 2), and that F(0)=0, f(1)=1, f(2)=1

(a) Show that there exists c1 in (0,1) such that f'(c1)=1

(b) Show that there exists c2 in (1,2) such that f'(c2)= 0.

(c) Show that there exists c in (0,2) such that f'(c)= 1/3