Multivariate Calculus

Ques 1) Find the following limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2}$$

Ques -2) At what points (x, y, z) in the plane $f(x, y) = \frac{y+1}{x^2+z^2-1}$ is continuous.

Ques-3) Show that the function $f(x, y) = \frac{x^2+y}{y}$ do not have the limit at the origin.

Ques 4)Find the points (x, y) in the plane $f(x, y) = \sqrt{x - y}$ is continuous.

Ques 5) Find the directional derivative of the function $f(x, y, z) = xyz$ in the direction of vector $v =< 5, -3, 2 >$.

Ques 6) Find the equation of the tangent plane to the surface $z = 4x^3y^2 + 2y$ at point $(1, -2, 12)$.

Ques 7) Let $f(x, y) = x \exp^{-y} + 5y$. Then find the slope of the surface $z = f(x, y)$ in the $x - direction$ at the point $(3, 0)$.

Ques 8) Show that if $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$,then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

Ques 9) The length and width of a rectangle are measured with error of at most 3% and 5% respectively. Use differentials to approximate the maximum percentage error in the calculated area.

Ques 10)Find the equation of the tangent plane to the surface $z = 10 - x^2 - y^2$ at P(2,,2,2).

Ques 11)Find the total differential of the function $f(x, y, z) = 3x^3 - 2y^4 + 5z$.

Ques 12) Find the directional derivative of the function $f(x, y) = \frac{e^{-x}}{y}$ at the point $(2, -1)$ in the direction of unit vector $\hat{u} = -\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$.

Ques 13) Let $f(x, y) = x^2 e^y$. Find the maximum value of a directional derivative at $(-2, 0)$ and find the unit vector in which the maximum value occurs.

Ques 14)Find a vector that is normal to the level surface $x^2+2xy+yz+3z^2 = 5$ at the point $P(1, 1, -1)$.

Ques 15) Find an equation for the tangent to the ellipse $x^2 + 4y^2 = 36$ at the point(0,3).

Ques 16) Evaluate $\frac{dz}{dt}$ using chain rule at the given value of t for the function

$$z = x^2 + y^2;\ x = cos\, t, y = sint, t = \pi$$

Ques 17) Using chain rule, find $\frac{dz}{dt}$ for $z = x - y, x = at, y = b\, cos\, at.$

Ques 18) For two independent parameters, apply chain rule for $f(x, y) = e^{xy}, x = u - v, y = u + v.$

Ques 19) Find $\frac{dw}{dt}$ where $w = \sin xyz$ and $x = 1 - 3t, y = e^{1-t}, z = 4t.$

Ques 20)Find the local extreme values of the function

$$f(x, y) = x^2 + xy + y^2$$

Ques 21) Find all critical points on the graph of $f(x, y) = 8x^3 - 24xy + y^3$ and classify the as a local extremum, or a saddle point.

Ques 22)Find the absolute maximum or minimum values of the function $f(x, y) = 2x^2 + y^2$ on the semicircle $x^2 + y^2 = 4, y \geq 0.$

Ques 23) A rectangular box with no top is to have a fixed volume. What should be its dimensions if we want to use the least amount of material in its construction.

Ques 24) Find the absolute maximum or minimum values of the function $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by lines $x = 0, y = 2, y = 2x$ in the first quadrant.

Ques 25) Describe the behaviour of the function $f(x, y) = 2\cos(x+y)+e^{xy}$ at the origin.

Ques 26) Use the method of Lagrange's multiplier to find the dimensions of a rectangle with perimeter p and maximum area.

Ques 27) Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

Ques 28) Find a point $P(x, y, z)$ closest to the origin on the plane $2x+y-z = 5$.

Ques 29)Evaluate the integral $\int_0^1 \int_0^2 (x^2 + xy + y^2) dx dy$.

Ques 30) Evaluate the integral $\int_0^1 \int_0^1 \frac{x}{(xy+1)^2} dy dx$.

Ques 31) Evaluate the double integral over rectangular region R

$$\int\int_R 4xy^2 dA;\ R = \{(x, y) : -1 \leq x \leq 1, -2 \leq y \leq 2\}$$

Ques 32) Evaluate the double integral over rectangular region R

$$\int\int_R \sin(x + y) dA;\ R = \{(x, y) : 0 \leq x \leq \frac{\pi}{4}, 0 \leq y \leq \frac{\pi}{2}\}$$

Ques 33) Find the volume of solid enclosed by the surface $z = x^2$ and the planes

$$x = 1, x = 2, y = 3, y = 0$$

Ques 34) Find the area of the region D which lies between $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and $x + y = a$.

Ques 35)Evaluate $\int_0^1 \int_0^y \sqrt{xy}\, dx dy$.