[ALGEBRA –II]

[Group Theory-I]

Q.1. Write out a complete multiplication Table for D_3

Q.2. Is D_3 Abelian ?

Q.3. In D_n ,explain geomertrically why rotation followed by a rotation must be a rotation .

Q.4. Show that {1,2,3,} under multiplication modulo 4 is not a group but that {1,2,3,4} under multiplication modulo 5 is a group.

Q.5. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G .

Q.6. Prove that if $(ab)^2=a^2b^2$ in a group G , then ab =ba.

Q.7. Prove that in any group, an element and inverse have the same order.

Q.8. Show that Z_{10}=<3>=<7>=<9>. Is Z_{10}=<2>?

Q.9. Find a group that contain elements a and b such that |a|=|b|=2 and

(a) |ab|=3 (b) |ab|=4 (c) |ab|=5

Can you see relationship b/w |a|,|b| and |ab|?

Q.10. Find a cyclic subgroup of order 4 in U(40).

Q.11. Find a non-cyclic subgroup of order 4 in U(40).

Q.12. Let G be a finite group with more than one element .Show that G has an element of prime order.

Q.13. Find all generators of Z_6,Z_8, and Z_{20}.

Q.14. List the elements of the subgroups (20) and (10) in Z_{30}.

Q.15. Find an example of a non-cyclic group, all of whose proper subgroups are cyclic.

Q.16. Suppose that a has infinite order. Find all generators of the subgroup (a^3).

Q.17. List the cyclic subgroups of U(30).

Q.18. Prove that a group of order 3 must be cyclic.

Q.19. Determine the subgroup lattice for Z_{12}.

Q.20. Let p be a prime and let G be an alelian group. Show that the set of all elements whose orders are powers of p is a subgroup of G.

Q.21. List all the element of Z_{40} That have order 10.

Q.22. Let $|x|=40$. list all the elements of $<x>$ that have order 10.

Q.23. Prove that , in any group , $|ab|=|ba|$.

Q.24. Prove that a group of order 4 is abelian .

Q.25. If p is an odd prime , prove that there is no group that has exactly p elements of order p.

Q.26. How many homomorphisms are there from Z_{20} onto Z_8.

Q.27. How many homomorphisms are there from Z_{20} onto Z_{10}.

Q.28. Supose that G is a finite group and that Z_{10} Is a homomorphic image of G. What can we say about $|G|$?

Q.29. Determine all homomorphic images of D_4.

Q.30. State and prove first isomorphism theorem.

Q.31. Prove that every group of order 65 is cyclic.

Q.32. Show that x^2+x+4 is a irreducible over Z_{11}.

Q.33. Prove that if W is a subspace of finite dimensional vector space v, then dim w+ dim w^0=dim v.

Q.34. Let H be a non-empty finite subset of a group G. Prove that H is a sub group of G if H is closed under the operation of G.

Q.35. Show that any infinite cyclic group is isomorphic to (Z,+).

Q.36. Find Aut (Z_{10}).