DIFFERENTIAL EQUATION-II

Q.1 Obtain the solution of the equation

(y-u)u_x+(u-x)u_y=x-y with the condition u=0 on xy=1.

Q.2 Show that the family of spheres

$x^2 + y^2 + (z - c)^2 = r^2$ satisfies the first order linear partial differential equation yp-xq=0.

Q.3 Find the general solution of the 1st order linear pde:

$$x\,u_x + yu_y = u.$$

Q.4 Find the general solution of linear equation:

$$(y - z)u_x + (z - x)u_y + (x - y)u_z = 0$$

Q.5 Obtain the solution of the equation

$xu_x + yu_y = x\exp(-u)$ with the data u=0 on $y = x^2$.

Q.6 Reduce each of the following equations to canonical form, and obtain general solution:

$u_x - u_y = u, \qquad yu_x + u_y = x$

Q.7 Show that the equation of motion of a long string is $u_{tt} = x^2 u_{xx} - g$, where g is gravitational acceleration.

Q.8 DeriveWave,Heat and Laplace equation in 2Dimension.

Q.9 Find the Characteristics,and Reduce to canonical form:

a) $u_{xx} + 2u_{xy} + 3u_{yy} + 4u_x + 5u_y + u = e^x$

b)$u_{xx} + yu_{yy} = 0$

Q.10 Find the solution of the Initial value problem

$$u_{tt} = c^2 u_{xx}, x \in R, t > 0,$$
$$u(x,0) = Sinx, u_t(x,0) = Cosx$$

Q.11 Determine the Solution of the Initial Boundary Value Problem

$$u_{tt} = 4u_{xx}, x > 0, t > 0,$$

$$u(x,0) = |Sinx|, x \geq 0,$$

$$u_t(x,0) = 0, x \geq 0$$

U(x,0)=0,t>0

Q 12.Using the successive approximation method

Find the first 3 approximations of sequence of functions that approaches the exact solution of the problem:

a) $\frac{dy}{dx} = xy, y(0) = 1$

b) $\frac{dy}{dx} = 1 + xy^2, y(0) = 0$

c) $\frac{dy}{dx} = e^x + y^2, y(0) = 0$

Q 13.Apply the Euler method to the initial value problem

$\frac{dy}{dx}$=2x+y, y(0)=1, using h=0.2,at x=0.2,0.4,0.6,0.8,1.0

Q 14. Apply the Modified Euler method to the initial value problem

$\frac{dy}{dx}$=2x+y, y(0)=1, using h=0.2,at x=0.2,0.4

Q15. Apply Runge-Kutta Method to IVP

$\frac{dy}{dx}$=2x+y, y(0)=1, using h=0.2,at x=0.2,0.4

Q 16. Find the General Solution of following:

$$\frac{dx}{dt} = 5x - 2y, \quad \frac{dy}{dt} = 4x - y$$

Q17. Consider the Linear System

$$\frac{dx}{dt} = 3x + 4y, \frac{dy}{dt} = 2x + y$$

Show that x=2e^{5t}, $x = e^{-t}$ *and* $y = e^{5t}$, $y = -e^{-t}$are solutions of this system

Q 18. Use the operator method to find the general solution of following:

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^{t}, \frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$