

Q 1. The following equation determine the constant A such that the equation is exact , and solve the resulting exact equation :

1. $(A x^2 y + 2 y^2) dx + (x^3 + 4xy) = 0$

2. $\left(\frac{A y}{x^3} + \frac{y}{x^2}\right)$

Q 2. Solve the initial value problem that consist of the differential equation

$$X \sin y dx + (x^2 + 1) \cos y dy = 0$$

And the initial condition $y(1) = \frac{\pi}{2}$

Q 3. Show that the homogeneous equation

$$(A x^2 + B xy + C y^2) dx + (D x^2 + E xy + F y^2) dy = 0$$

Is exact if and only if $B = 2D$ and $E = 2C$

Q 4. Solve $x'' + 2 x' + x = 0$ with $x(0) = 1$ and $x'(0) = 0$.

Q 5. Solve the given differential equation

1. $\frac{dy}{dx} + \frac{3y}{x} = 6 x^2$

2. $(y \sin 2x - \cos x) dx + (1 + \sin^2 x) dy = 0$

Q 6. In given equation find the integrating factor of the form $x^p y^q$ and solve

$$(4xy^2 + 6y) dx + (5x^2 y + 8x) dy = 0$$

Q 7. Solve the differential equation by making a suitable transformation .

$$(10x - 4y + 12) dx - (x + 5y + 3) dy = 0$$

Q 8. Solve $x'' - x = 0$ with $x(0) = x_0$, $x'(0) = u_0$.

Q 9. Find the general solutions of the following differential equations.

1. $y' = -2xy$ 2. $\frac{dy}{dx} = 1+x+y+xy$

Q 10. A certain city had a population of 25000 in 1960 and a population of 30000 in 1970.

Assume that its population will continue to grow exponentially at a constant rate.

What population can its city planners expect in the year 2000.

Q 11. Solve $x'' + x' + x = 0$ with $x(0) = 1$ and $x'(0) = 0$.

Q 12. When sugar is dissolved in water, the amount A that remains undissolved after t minutes satisfies the differential equation $\frac{dA}{dt} = -kA$ ($k > 0$). If 25% of the sugar dissolves after 1

min, how long does it take for half of the sugar to dissolve

- Q 13. Suppose that the growth of a population of a colony of bacteria is proportional to the number present. It is known that initially the population was 1000 and it gets doubled after 1 hour. Find the population of colony of bacteria after 1.5 hours.
- Q 14. Solve $x'' + 5x' + 6x = 0$ with $x(0) = 2$, $x'(0) = 3$,
- Q 15. Find a particular solution of $x'' - 3x' - 4x = 2 \sin t$.
- Q 16. Solve $x' + 2x = e^{-t}$ with $x(0) = \frac{3}{4}$
- Q 17. Find the particular solution of $y'' - 4y = 2e^{3x}$
- Q 18. Find the general form of particular solution of $y''' + 9y' = x \sin x + x^2 e^{2x}$
- Q 19. Verify that the function $y(x) = 2x^{1/2} - x^{1/2} \ln x$ satisfies the differential equation
$$4x^2 y'' + y = 0 \text{ for all } x > 0$$
- Q 20. Solve the initial value problem $\frac{dy}{dx} = 2x + 3$; $y(1) = 2$
- Q 21. Let in a lake, the pollution level is 5%. If the fresh water at the rate of 10000 litres per day is allowed to enter and same amount of water leaves the lake. Find the time when pollution level is 2.5% if volume of lake is 500000 litres. Further, if safety level is 0.1%, then after how much time, water is suitable for drinking.
- Q 22. Let in a lake, the pollution level is 7%. If the concentration of incoming water is 2% and 10000 liters per day water is allowed to enter the lake, find time when pollution level is 5%. Volume of the lake is 200000 liters. Also, find pollution after 32 days.
- Q 23. In a population, the initial population is 100. Suppose the population can be modeled using the differential equation $\frac{dx}{dt} = 0.2x - 0.001x^2$ with a time step of one month. Find predicted population after 2 months.
- Q 24. When sugar is dissolved in water, the amount A that remains undissolved in water after 't' min satisfies $dA/dt = -k$ ($k > 0$). If 25% sugar dissolves after 1 min how long does it take for half of the sugar dissolve.
- Q 25. A population, initially consisting of two mice has reproduction rate 1 mice/month/mouse.

Also 9/10 mouse traps are set each month is filled $k=10$. Find mouse population after 3 months.

Q 26. In mid 1999, total population of the world is 6 billion persons. It then increase at the rate of about 212 thousand persons each day.

- What is the annual growth rate k .
- What will be the world's total population at the middle of 21st century?
- How long will it take the world population to increase tenfold?

Q 27. A hemispherical bowl of radius 4ft is full of gasoline initially. A circular bottom hole with diameter 1 inch is opened. How long will it take for all the gasoline to drain the hole.

Q 28. Solve the initial value problem

$$(3x - y - 6)dx + (x + y + 2)dy = 0, \quad y(2) = -2$$

Q 29. Show that the homogeneous equation

$$(Ax + By)dx + (Cx + Dy)dy = 0 \quad \text{Is exact if and only if } B=C$$

Q 30. Solve the differential equation

$$2r(s^2 + 1)dr + (r^4 + 1)ds = 0$$

Q 31. Determine whether or not the given equation is exact:

$$(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6)dy = 0$$

Q 32. Use the method of variation of parameters to find a particular solution of the given differential equation

$$Y'' + 4y = \cos 3x$$

Q 33. Find the particular solution of

$$Y^{(5)} + 5Y^{(4)} - Y = 17$$

Q 34. Solve the initial value problem

$$Y''' + 3Y'' - 10Y' = 0$$

$$Y(0) = 7, \quad Y'(0) = 0, \quad Y''(0) = 70$$

Q 35. Find an integrating factor of the form $x^p y^q$ and solve

$$(4xy^2 + 6y) dx + (5x^2y + 8x)dy = 0$$