GE

Calculus

Ques-1) Calculate the volume of the solid generated by revolving the plane region bounded by $y = \sqrt{x}, x = 4, y = 1/2$ about the x-axis.

Ques-2) Compute the volume of the solid generated by revolving the plane region bounded by $y = x^2, y = 9$, and $x = 0$ about the y-axis.

Ques-3) Use the slicing method to find the volume of the solid generated by revolving the plane region bounded by $y = x^2$ and $y = 3$ about the line $y = -1$.

Ques-4) The plane region below $y = 1/x$, above $y = 0$, and to the right of $x = 1$ is revolved about the x-axis. Calculate the volume of the generated solid.

Ques-5) A pyramid has a triangular base of area A and has a height of h measured perpendicular to the plane of the base. Show that its volume is V = (1/3)Ah.

Ques-6) Calculate the volume of the solid generated by revolving the plane region bounded by y = 1/x, x = 1, and x = 3 about the x-axis.

Ques-7) A cylindrical hole of radius r is drilled through the centre of a ball of radius R. Compute the volume of the remaining part of the ball.

Ques-8) Find this limit if it exists $\lim_{x\to 3} \frac{x^2-9}{x^2-3x}$.

Ques-9) Use the shell method to find the volume of the solid generated by revolving the plane region bounded by $y = x^2, y = 9$, and x = 0 about the y-axis.

Ques-10) The plane region bounded by $x = y^2$ and $y = -x + 2$ is revolved about the line $y = 1$. Find the volume of the generated solid by using the shell method.

Ques-11) Sketch the graph of the function:

$$\frac{x^3 - 4x}{x^2 - 1}$$

Ques-12) Sketch the graph of the function $y = x^2(x^2 - 1)$, making use of any suitable information you can obtain from the function and its first and second derivatives.

Ques-13) Sketch the graph of the function:

$$y = \frac{2x+4}{x-1}$$

using information from the function and its first and second derivatives.

Ques-14)Sketch the graph of the function:

$$y = \frac{x^3}{x-1}$$

employing any useful information you can get from $y, y', andy''$.

Ques-15) Sketch the graph of a twice differentiable function f where $f''(x) < 0$ for $x < 2, f(2) = -1, f'(2) = 1, f''(2) = 0, f''(x) > 0$ for $x > 2$, and $f(4) = 3$.

Ques-16) Let

$$f(x) = \frac{x+2}{x^3 - 7x + 6}$$

Where is f continuous and where is it discontinuous?

Ques-17) Let [x] denote the greatest integer less than or equal to x.
a) Sketch a graph of y = [x].
b) Where is [x] continuous and where is it discontinuous?

Ques-18) Find any horizontal and vertical asymptotes of $f(x) = \frac{3}{x-2}$.

Ques-19) Prove that there's a point on the line:

$$y = \frac{2-3x}{\sqrt{3}}$$

that's closest to the origin. Find that point.

Ques-20) Let $f(x) = a_n x_n + a_{n-1} x_{n-1} + a_{n-2} x_{n-2} + ... + a_1 x + a_0$, where n is an even positive integer and $a_n > 0$. Prove that f attains a minimum over the entire real line.

Ques-21) Prove that:

$$\lim_{x \to 9} \sqrt{x} = 3$$

utilizing the definition of the limit.

Ques-22) Find the equation of the tangent plane to the surface $z = 10 - x^2 - y^2$ at P(2,,2,2).

Ques-23)Find the directional derivative of the function $f(x, y) = \frac{e^{-x}}{y}$ at the point $(2, -1)$ in the direction of unit vector $\hat{u} = -\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$.

Ques-24)Find a vector that is normal to the level surface $x^2+2xy+yz+3z^2 = 5$ at the point $P(1, 1, -1)$.

Ques-25)Find an equation for the tangent to the ellipse $x^2 + 4y^2 = 36$ at the point(0,3).

Ques -26) Evaluate $\frac{dz}{dt}$ using chain rule at the given value of t for the function

$$z = x^2 + y^2; \, x = \cos t, y = \sin t, t = \pi$$

Ques -27) Using chain rule, find $\frac{dz}{dt}$ for $z = x - y, x = at, y = b \cos at$.

Ques -28)Find the local extreme values of the function

$$f(x, y) = x^2 + xy + y^2$$

Ques -29) Find all critical points on the graph of $f(x, y) = 8x^3 - 24xy + y^3$ and classify the as a local extremum, or a saddle point.

Ques -30)Find the absolute maximum or minimum values of the function $f(x, y) = 2x^2 + y^2$ on the semicircle $x^2 + y^2 = 4, y \geq 0$.

Ques- 31) A rectangular box with no top is to have a fixed volume. What should be its dimensions if we want to use the least amount of material in its construction.

Ques-32) Find the absolute maximum or minimum values of the function $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by lines $x = 0, y = 2, y = 2x$ in the first quadrant.

Ques-33) Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

Ques-34) Find a point $P(x, y, z)$ closest to the origin on the plane $2x+y-z = 5$.

Ques-35) Decide on the convergence or divergence of $\int_1^\infty \frac{\sin^2 \frac{1}{x}}{\sqrt{x}} dx$.