QUESTION OF ALGEBRA

Q 1. Find the sixth roots of complex number (1+i) and represent them in the complex plane.

Q 2. Prove that the intervals (2,5) and (10,∞) have same cardinality.

Q 3. Solve the equation :

$$z^{10} + (-2+i)z^5 - 2i = 0$$

Q 4. if :

u_1=(1,-3,2) , u_2=(2,-4,-1) , u_3=(1,-5,7) , u_4= (2,-5,3) belongs to R^3.

Check whether $u_4 \in \{u_1 , u_2 , u_3\}$ or not . Is $\{u_1 , u_2 , u_3 , u_4\}$

Linearly dependent or linearly independent ?

Q 5. Define linear independence of n vectors . How many pivot columns must a 7× 5 matrix have if its columns are linearly independent ? Why ?

Q 6. Express Sin 6θ in powers of Sin θ and Cos θ .

Q 7. If a=bq +r for integers a , b , q and r then prove that g.c.d. (a ,b) = g.c.d. (b ,r)

Q 8. use the principle of mathematical induction to prove that n! > n^3 ∀ n ≥ 6 .

Q 9. Consider the following system of linear equations :

a+b-2c+4d = 5

2a+2b-3c+d = 3

3a+3b-4c-2d = 1

(i) Write the matrix equation and the vector equation of the above system of equations .

(ii) Find the general solution in parametric vector form by reducing it into Echelon form .

(iii) List the pivot columns .

Q 10. For a ,b ϵ Z , Define a~b if and only if $a^2 - b^2$ is divisible by 3.

(a) Prove that ~ *defines an equivalence relation on Z.*

(b) What is $\overline{0}$ and $\overline{1}$?

Q 11. Find x such that $(1080)^5 \equiv x(\text{mod } 7)$.

Q 12. Define Projection mapping from $R^2 \rightarrow R^2$ on x – axis and y – axis .

Q 13. Show that the linear transformation T : $R^3 \rightarrow R^3$ given by

T (a, b, c) = (a+b, b+c, a+c)

Is one –one and onto .

Q 14. Define subspace of R^n . Let H = { (a, b, c) ϵ R^3 | c = 2a+b }

(i) Show that H is a subspace of R^3 .

(ii) Prove that the Eigen values of a triangular matrix are the entries on the main diagonal .

Q 15. Find the geometric image of the complex number z , where :

$$|z+i| \geq 2 .$$

Q 16. Let T : $R^n \rightarrow R^m$ be a linear transformation . Then prove that T is one – one if and only if the equation T(x) = 0 has only the trivial solution .

Q 17. Find the basis and dimension for Column space of A and Null space of A where

$$A = \begin{pmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence find rank of A .

Q 18. Find the general solution of linear system whose augmented matrix has been reduced to

Probability

$$\begin{pmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{pmatrix}$$

Q 19. The diagonals of a square intersect at A and B is one vertex of the square. Prove that the four vertices of the square are represented by the numbers;

a +(b-a), a +i(b-a), a −(b-a), a −i(b-a),

Where the point A, B are represented by the complex numbers a, b.

Q 20. Define congruence modulo n, where n is a positive integer. Prove that it is an equivalence relation. Find all congruence classes of integers modulo 5

Q 21. Determine the values of the constants a, b, c in the equation;

$2^4\, Cos^5 x$ = a Cos 5x + b Cos 3x + c Cos x

Q 22. Using D escartes rule of signs, show that the equation :

$x^6 - 2x^5 + x^4 + x^2 - 2x + 1 = 0$ must have at least two complex root.

Q 23. If α, β, γ are the roots of the equations

$x^3 + ax^2 + bx + a = 0$ where a and b are real then show that

$Tan^{-1}\alpha + Tan^{-1}\beta + Tan^{-1}\gamma = n\pi$ except when b = 1.

Q 24. Sum the series Cos x + Cos(x+y) + Cos (x+2y) + ... + up to n terms.

Q 25. Using De – Moivre's theorem solve the equation

$z^5 + z^4 + z^3 + z^2 + z + 1 = 0$

Q 26. Define the rank of a matrix. Reduce the matrix A = $\begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$ to normar form using elementary operations and hence determine its rank.

Q 27. If A and P are square matrices of the same order and P is invertible, show that A and P^{-1} A P have the same characteristic roots.

Q 28. For what values of λ does the following system of equations have a solution

x + y + z = 1

Probability

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

also find the solution in each case

Q 29. Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$

Given that the sum of two roots is equal to the sum of other two roots

Q 30. Prove that an elementary row operation on the product of two matrices is equivalent to the same elementary row operation on the pre factor .

Q 31. Solve the equation $x^4 + 12x^3 + 49x^2 - 78x + 40 = 0$ by removing its second term .

Q 32. If $x + y + z = 1$

$x^2 + y^2 + z^2 = 2$

$x^3 + y^3 + z^3 = 3$

Find the value of $x^4 + y^4 + z^4$

Q 33. Solve the congruence $8x \equiv 5 \pmod{27}$

Q 34. Let : $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{pmatrix}$

(a) Find Eigen Values of A.

(b) Find Eigen vectors and eigen space corresponding to each eigen value .

Q 35. Find the basis and dimension for Column space of A and null space of A where :

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix}$$

Hence find Rank of A .