MATHEMATICS FINANCE

Q:1:-How much interest do you get if you put 1000 pounds for two years in a savings acount that pays simple interest at a rate of 9% per annum? And if you leave it in the account for only half ar year?

Answer. If you leave it for two years, you get $2 \cdot 0.09 \cdot 1000 = 180$ pounds in interest. If you leave it for only half a year, then you get 1 2 $\cdot 0.09 \cdot 1000 = 45$ pounds. As this example shows, the rate of interest is usually quoted as a percentage; 9% corresponds to a factor of 0.09. Furthermore, you have to be careful that the rate of interest is quoted using the same time unit as the period. In this MATH1510 1 example, the period is measured in years, and the interest rate is quoted per annum ("per annum" is Latin for "per year"). These are the units that are used most often. In Section 1.5 we will consider other possibilities.

Q:2:-How much do you have after you put 1000 pounds for two years in a savings acount that pays compound interest at a rate of 9% per annum? And if you leave it in the account for only half ar year?

Answer. If you leave it in the account for two years, then at the end you have $(1 + 0.09)2 \cdot 1000 = 1188.10$, as we computed above. If you leave it in the account for only half a year, then at the end you have $(1 + 0.09)1/2 \cdot 1000 = \sqrt{} 1.09 \cdot 1000 = 1044.03$ pounds (rounded to the nearest penny). This is 97p less than the 45 pounds interest you get if the account would pay simple interest at the same rate.

Q:3:- Suppose that the interest rate is 7%. What is the present value of a payment of e70 in a year's time?

Answer. The discount factor is $v = 1/1.07 = 0.934579$, so the present value is $0.934579 \cdot 70 = 65.42$ euro (to the nearest cent). Usually, interest is paid in arrears. If you borrow money for a year, then at the end of the year you have to pay the money back plus interest. However, there are also some situations in which the interest is paid in advance. The rate of discount is useful in these situations.

Q:4:- Compare the following three loans: a loan charging an annual effective rate of 9%, a loan charging 8 3 4% compounded quarterly, and a loan charging 8 1 2% payable in advance and convertible monthly.

Answer. We will convert all rates to annual effective rates. For the second loan, we use (1.5) with $p = 4$ and $i (4) = 0.0875$ to get $1+i = (1+i (p)/p) p = 1.0904$, so the annual effective rate is 9.04%. For the third loan, we use (1.7) with $p = 12$ and $d (12) = 0.085$ to get $1 - d = (1 - d (p)/p) p = 0.91823$. Then, we use (1.1) and (1.3) to deduce that $1 + i = 1 v = 1 /1-d = 1.0890$, so the annual effective rate is 8.90%. Thus, the third loan has the most favourable interest rate.

Q:5:-We computed that a loan of e2500 at 6 1 2% interest can be repaid by ten installments of e347.76, each being paid at the end of the year. What is the remaining balance of the loan after six years?

Answer. There are two methods to handle questions like this. The first method considers the payments in the first six years. This is called the retrospective method, because it looks back to payments already made. The second method considers the payments in the last four years. This is called the prospective method, because it looks forward to payments that have not been made yet. Obviously, both methods should give the same answer, and you should pick the method that seems more convenient. The retrospective method uses that the remaining balance is the value of the original loan after six years minus the accumulated value of the payments that have already been made. The borrower has made six payments of e347.76 each at the end of the year. The accumulated value of these payments is

$347.76 \cdot s6 = 347.76 \cdot 7.063728 = 2456.48.$

The value of the loan after six years is

$2500 \cdot (1 + i) 6 = 2500 \cdot 1.459142 = 3647.86,$

so the remaining balance of the loan is 3647.86 − 2456.48 = 1191.38 euros. The prospective method uses that the remaining balance equals the present value of the remaining payments. The borrower still has to make four payments of e347.76. We need the present value of these payments six years after the start of the loan. This is one year before the first of the four remaining payments is due, so the present value of the four remaining payments is $347.76 \cdot a4 = 347.76 \cdot 3.425799 = 1191.36$ euros. Thus, the remaining balance of the loan after six years is e1191.36. The results found by the retrospective and prospective methods differ by two cents. The difference is caused because at the end of Example 2.1.4, the value of 347.7617 . . . was rounded to 347.76. The prospective and retrospective method would have given the same result if we had used the exact value.

Q:6:- An investor is contemplating two investment projects. Project A requires an initial payment of £10000, in return for which the investor will receive £250 at the end of every quarter for 15 years. Project B requires an initial payment of £11000. In return, the investor will be paid £605 at the end of every year for 18 years and the initial payment of £11000 will be repaid at the end.

Both projects have one outlay at the start, and payments to the investor afterwards, so they both have a well-defined internal rate of return. The net present value for Project A is given by NPVA(i) = −10000 + 1000a (4) 15 .

The yield is found by solving NPVA(i) = 0, or a (4) 15 = 10, resulting in a yield of (approximately) 5.88% p.a. On the other hand, the net present value for Project B is NPVB(i) = −11000 + 605a18 + 11000v 18 . The internal rate of return for Project B is 5.5% p.a. The net present values of both projects are plotted in Figure 3.3. This shows that if the interest rate is low enough, Project B is more profitable than Project A, even though it has a lower yield. The rate at which the graph cross, that is, the rate i at which NPVA(i) = NPVB(i), is called the cross-over rate. In this example, the cross-over rate is approximately 5.11%. If the investor can borrow money for a lower rate than the cross-over rate, he will make a larger profit on Project B than on Project A. For instance, if the investor may lend or borrow money at 4%, then the profit on Project A is NPVA(0.04) = 1283.81, while the profit on Project B is NPVB(0.04) = 2088.82.

Q:7:-The annual rate of compound interest equals 8%. After how many years will the original sum double?

Solution. An inequality must be solved: (1 0.08)n+> 2. Let us take the logarithm based on the natural logarithms and obtain n >ln(2)/ln(1.08). Answer: after 9 years.