LINEAR PROGRAMMING

Q:1:-Find the maximum value of z to the following LPP using graphical method,

$Max z = 5x_1 + 7x_2$, subject to $x_1 + x_2 \leq 4$, $3x_1 + 8x_2 \leq 24$, $10x_1 + 7x_2 \leq 35$ $and x_i \geq 0$, $\forall i = 1,2$.

Solution:

The given problem contains only two variables x_1 *and* x_2. So graphical method is possible. Consider the equations, we get

$$x_1 + x_2 = 4,$$

$$3x_1 + 8x_2 = 24,$$

$$10x_1 + 7x_2 = 35$$

Now to find the points,

Consider $x_1 + x_2 = 4$,

Put $x_1 = 0 \Rightarrow x_2 = 4$

and $x_2 = 0 \Rightarrow x_1 = 4$

Therefore (0,4) and (4.0) are the pts.

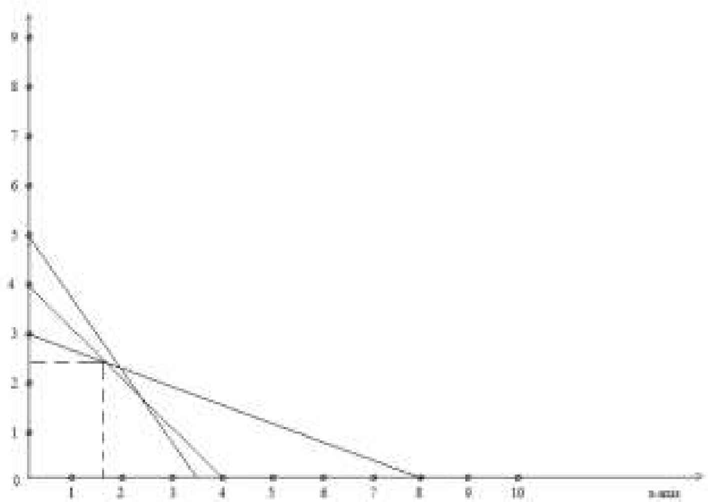
Similarly consider $3x_1 + 8x_2 = 24$,

We get (0,3) and (8,0) are the pts.

Consider $10x_1 + 7x_2 = 35$.

We get (0,5) and (3.5,0) are the pts.

The graphical representation is



Solution is $x_1 = 1.6$, $x_2 = 2.4$ and $Max\ z = 24.8$

Q:2:- Solve the following LPP using simplex method, $Max z = 5x_1 + 3x_2$, subject to $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$ $and x_i \ge 0$, $\forall i = 1,2$.

The objective function is maximization so the resulting LPP becomes

$Max\ z = 5x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$ subject to

$$x_1 + x_2 + x_3 = 2$$

$$5x_1 + 2x_2 + x_4 = 10$$

$$3x_1 + 8x_2 + x_5 = 12, x_i \ge 0, \forall i = 1,2\ldots5$$

The initial basic feasible solution is obtained by putting $x_1 = x_2 = 0$ in the reformulated form of LPP and we get $x_3 = 2, x_4 = 10, x_5 = 12$.

<u>Starting Simplex Table:</u>

		C_j	5	3	0	0	0	Ratio
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4	Y_5	$\frac{X_{Bi}}{Y_{ir}}$
0	X_3=2	Y_3	1	1	1	0	0	2
0	X_4=10	Y_4	5	2	0	1	0	2
0	X_5=12	Y_5	2	8	0	0	1	4
	$z_j = \sum_{j=1}^{n} C_{Bi} Y_{ij}$		0	0	0	0	0	
	$z_j - c_j$		-5	-3	0	0	0	

First Iteration:

New Pivot equation=$\frac{old\ \ equation}{pivot\ \ element}$

		C_j	5	3	0	0	0
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4	Y_5
5	X_1=2	Y_1	1	1	1	0	0
0	X_4=0	Y_4	0	-3	-5	1	0
0	X_5=6	Y_5	0	5	-3	0	1
	$z_j = \sum_{j=1}^{n} C_{Bi} Y_{ij}$		5	5	5	0	0
	$z_j - c_j$		0	2	5	0	0

Hence an optimal basic feasible solution is obtained.

Solution is $x_1 = 2,\ x_2 = 0$ and $Max\ z = 10$

Q:3:- Solve the following LPP using Big-M method, $Min z$ = 12x_1 + 20x_2 , subject to 6x_1 + 8x_2 ≥ 100, 7x_1 + 12x_2 ≥ 120,$and x_i$ ≥ 0, ∀i = 1,2.

The objective function is converted to maximisation objective function.

The resulting LPP becomes,

$Max(-z) = -12x_1 - 20x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$ subject to

$$6x_1 + 8x_2 - x_3 + x_5 = 100$$

$$7x_1 + 12x_2 - x_4 + x_6 = 120, x_i \geq 0, \forall\ i.$$

The initial basic feasible solution is obtained by putting $x_1 = x_2 = x_3 = x_4 = 0$ in the reformulated form of LPP and we get $x_5 = 100, x_6 = 120.$

Starting Simplex Table:

		C_j	-12	-20	0	0	-M	-M	Ratio
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	$\frac{X_{Bi}}{Y_{ir}}$
-M	X_5=100	Y_5	6	8	-1	0	1	0	12.5
-M	X_6=120	Y_6	7	12	0	-1	0	1	10
	$z_j = \sum_{i=1}^{n} C_{Bi} Y_{ij}$		-13M	-20M	M	M	-M	-M	
	$z_j - c_j$		-13M+20	-20M+20	M	M	0	0	

First Iteration:

		C_j	-12	-20	0	0	-M	Ratio
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4	Y_5	$\frac{X_{Bi}}{Y_{ir}}$
-M	X_5=20	Y_5	4/3	0	-1	2/3	1	15
-20	X_2=10	Y_2	7/12	1	0	-1/12	0	120/7
	$z_j = \sum_{i=1}^{n} C_{Bi} Y_{ij}$		-4M/3-35/3	-20	M	-2M/5+5/3	-M	
	$z_j - c_j$		-4M/3+1/3	0	M	-2M/5+5/3	0	

Second Iteration:

		C_j	-12	-20	0	0
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4
-12	X_1=15	Y_1	1	0	-3/4	½
-20	X_2=5/4	Y_2	0	1	7/16	-5/12
	$z_j = \sum_{i=1}^{n} C_{Bi} Y_{ij}$		-12	-20	¼	7/3
	$z_j - c_j$		0	0	¼	7/3

Hence an optimal basic feasible solution is obtained.

Solution is $x_1 = 15$, $x_2 = 5/4$ and $Max(-z) = -205$

Q:4:- Solve the following LPP using Dual Simplex method,

$Min z$ = 3x_1 + x_2 , subject to x_1 + x_2 ≥ 1, 2x_1 + 3x_2 ≥ 2, x_1 , x_2 ≥ 0.

The objective function is a minimization problem so convert it into a maximisation problem. The resulting LPP becomes,

$Max\ z^* = -3x_1 - x_2 + 0x_3 + 0x_4$ subject to

$$-x_1 - x_2 + x_3 = -1$$

$$-2x_1 - 3x_2 + x_4 = -2$$

$$x_i \geq 0, \forall\ i = 1,2,3,4.$$

The initial basic feasible solution is obtained by putting $x_1 = x_2 = 0$ in the reformulated form of LPP and we get $x_3 = -1, x_4 = -2$.

Starting Simplex Table:

		C_j	-3	-1	0	0
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4
0	X_3=-1	Y_3	-1	-1	1	0
0	X_4=-2	Y_4	-2	-3	0	1
	$z_j = \sum_{i=1}^{n} C_{Bi} Y_{ij}$		0	0	0	0
	$z_j - c_j$		3	1	0	0

For calculating the entering variable we find out the replacement ratio

By max$\left(\frac{z_j - c_j}{Y_{kj}}, Y_{kj} < 0\right)$=max(-3/2, -1/3, 0/0, 0/1)= -1/3.

First Iteration:

		C_j	-3	-1	0	0
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4

0	X_3=-1/3	Y_3	-1/3	0	1	-1/3
-1	X_2=2/3	Y_2	2/0	1	0	-1/3
	$z_j = \sum_{j=1}^{n} C_{Bi} Y_{ij}$		-2/3	-1	0	1/3
	$z_j - c_j$		7/3	0	0	1/3

Calculate the ratio max($\frac{z_j - c_j}{Y_{kj}}, Y_{kj} < 0$)=max(-7, 0/0, 0/1, -1)= -1.

<u>Second Iteration:</u>

		C_j	-3	-1	0	0
C_B	X_B	Y_B	Y_1	Y_2	Y_3	Y_4
0	X_4=1	Y_4	1	0	-3	1
-1	X_2=1	Y_2	-1/3	1	-1	0
	$z_j = \sum_{j=1}^{n} C_{Bi} Y_{ij}$		1/3	-1	1	0
	$z_j - c_j$		10/3	0	1	0

Hence an optimal basic feasible solution is obtained.

Solution is $x_1 = 0,\ x_2 = 1$ and $Max\ z^* = -1$

Min z= - $Max\ z^*$= -(-1)=1

Q:5:- Find an initial basic feasible solution to the following TP using North-West Corner Rule.

	1	2	3	4	a_i
1	10	0	20	11	15
2	12	7	9	20	25
3	0	14	16	18	5
b_j	5	15	15	10	45

Solution:

Step 1: First allocate in (1,1) cell. x_{11}=min(15,5)=5

a_i	1	2	3	4	
1	10	0	20	11	15
2	12	7	9	20	25
3	0	14	16	18	5
b_j	0	15	15	1	45

Step 2: Second allocate in (1,2) cell. x_{12}=min(10,15)=10

	2	3	4	
~~1~~	~~0~~	~~20~~	~~11~~	~~10~~
2	7	9	20	25
3	14	16	18	5
	5	15	10	

Step 3: Third allocate in (2,2) cell. X_{22}=min(25,5)=5

	2	3	4	
2	7	9	20	25
3	14	16	18	5
	5	15	10	

Step 4: Fourth allocate in (2,3) cell. X_{23}=min(20,15)=15

	3	4	
2	9	20	20
3	16	18	15
	15	10	

Step 5: Fifth allocate in (2,4) cell. X_{24}=min(5,10)=5

	4	
~~2~~	~~20~~	~~0~~
3	18	5
	10	

Step 6: Sixth allocate in (3,4) cell. X_{34}=min(5,5)=5

	4	
3	18	5
	5	

The initial basic feasible solution is given by $x_{11} = 5, x_{12} = 10, x_{22} = 5,$

$$x_{23} = 15, x_{24} = 5, x_{34} = 5$$

	1	2	3	4
1	5 10	10 0	20	11
2	12	5 7	15 9	5 20
3	8	14	16	5 18

Minimum transportation cost=5*10+10*0+5*7+15*9+5*20+5*18=Rs.410

Q:6:- A shop has 4 workers and 4 works to be performed. The estimate of time(man hours) each man will take to perform each project is given in the following table. How should the works to be allotted so as to optimise the total work.

	1	2	3	4
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

Solution:

Step 1: Given matrix is,

	1	2	3	4
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

Step 2: Here the given matrix is a square matrix. Hence the assignment problem is a balanced problem. Subtracting minimum cost of each row of the cost matrix we obtain the following matrix.

	1	2	3	4
1	0	18	9	3
2	9	24	0	22
3	23	4	3	0
4	9	16	14	0

Step 3: Subtracting minimum cost of each column of the modified cost matrix we obtain the following matrix.

	1	2	3	4
1	0	14	9	3
2	9	20	0	22

3	23	0	3	0
4	9	12	14	0

Step 4: Drawing the minimum possible number of horizontal and vertical lines.

	1	2	3	4
1	0	14	0	3
2	9	20	0	22
3	23	0	3	0
4	9	12	14	0

Here the number of lines=4=order of cost matrix.Hence optimum assignment obtained.

	1	2	3	4
1	0			
2			0	
3		0		
4				0

Optimum Schedule is 1->1, 2->3, 3->2, 4->4

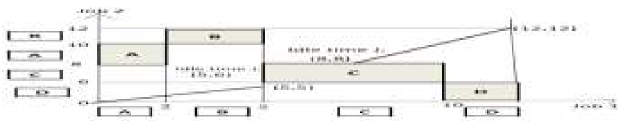
Minimum assignment cost=8+4+19+10=41.

Q:7:- Use graphical method to find the minimum total elapsed time needed to process the following two jobs on four machines A,B,C,D given the processing times and the sequ-ences.

Job 1	*Sequence:* *Time in Hrs:*	*A* *2*	*B* *3*	*C* *5*	*D* *2*
Job 2	*Sequence:* *Time in Hrs:*	*D* *6*	*C* *2*	*A* *3*	*B* *1*

Solution:

The graph representing the data is given below. The rectangular blocks in the graph are obtained by pairing the same machines. The path from the origin to the end point, which uses maximum diagonal segments, is shown in the figure. For other details the figure is self- explanatory.



The minimum total elapsed time = Processing time for Job 1+ idle time for Job 1

=12+3=15 hours (or)

The minimum total elapsed time = Processing time for Job 2+ idle time for Job 2

=12+3=15 hours

Q:8:-What is Queueing System.

A queuing system can be completely described by

(a) The input (or arrival pattern),

(b) The service mechanism (or service pattern)

(c) The „ queue discipline „ and

(d) Customer‟sbehaviour

(a) The input (or arrival pattern). The input describes the way in which the customers arrive and join the system. Generally, the customers arrive in a more or less random fashion which is not worth making the prediction. Thus, the arrival patteren can best be described in terms of probabilities and consequently the probability distribution for inter – arrival times (the time between two successive arrivals) or the distribution of number customers arriving in unit time must be defined.

(b) The service mechanism (or service pattern). It is specified when it is known how many customers can be served at a time, what the statistical distribution of service time is, and when service is available. It is true in most situations that service time is a random variable with the same distribution for all arrivals, but cases occur where there are clearly two or more classes of customers, (e.g., machines waiting repair) , each with a different service time distribution. Service time which are important in practice are the „ negative exponential distribution‟ and the related „ Erlang(

gamma) distribution „. Queues with the negative exponential service time distribution are studied in the following sections.

(c) The queue discipline. The queue discipline is the rule determining the formation of the queue, the manner of the customer‟sbehavior while waiting, and the manner in which they are chosen for service. The simplest discipline is " First come, first served", according to which the customers are served in the order of their arrival. For example, such type of queue discipline is observed at a ration shop, at cinema ticket windows, at inrailway stations, etc., If the order is reversed we have the " last come, first served " discipline, as in the case of a big go down the items which come last or taken out first. An extremely difficult queue discipline to handle might be " service in random order " or " might is right ".

Notations:

FIFO ⟹ First In, First Out

FCFS ⟹ First come, First Served

LIFO ⟹ Last In, First Out

SIRO ⟹Service in Random Order

FILO⟹ First In, Last Out.

(d) Customer's behavior. The customers generally behave in four ways:

(i) Balking . A customer may leave the queue because the queue is too long and he has no time to wait, or there is not sufficient waiting space.

(ii) Reneging . This occurs when a waiting customer leaves the queue due to impatience.

(iii) Priorities. In certain applications some customers are served before others regardless of their order of arrival. There customers have priority over others.

(iv) Jockeying. Customers may jockey from one waiting line to another. It may be seen that this occurs in the supermarket.

Q:9:-What is Inventory Control. What are the reason for holding stocks. Write objective of Inventory Control

A inventory may be defined as an idle resources that possesses economic value. It is an item stored or reserved for meeting future demand. Such items may be materials machines, many or even human resources.

Reasons for holding storks

The main reasons are

(1)To ensure sufficient goods one are available to meet anticipated demands.

(2)To absorb variations in demand and production.

(3)To prove a buffer between production processor.

(4)To take advantage of bulk purchasing discounts.

(5)To meet possible shortages in the future.

(6)To enable produces process to flow smoothly and efficiently

(7)As deliberate investment policy particularly in times of inflation or possible shortages.

The Objective of Inventory Control

The objective of inventory control is to maintain stock levels so that the combined costs, mentioned earlier are at amenities. This is done by establishing two sectors. “how to order? When to order?”Inventory control terminologies.

(1)Demand : The amount of quantity required by sales or products usually expressed as the rate of demand for week or month or year etc.

(2)Economic order quantity (EOQ) This is a calculated ordering quantity which minimized the balance between inventory holding costs and re-order costs.

(3)Lead time: The period of time between ordering and replenishment.

(4)Butter stock (or safety stock). It is a stock allowance to cover errors in forecasting the lead time on the demand during the lead time.

(5)Maximum stock : A stock level as the maximum desirable which is used as an indicator.

(6)Reorder level: The level of stock of which culture replacement order should be placed. The re-order level 9 independent upon the lead time

Q:10:- A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and storage cost amounts to 60paise per unit per unit per year. The set up cost associated with ordering 200% higher than EOQ.

D = 600 units per year

Cs= Rs.80 per setup

C_1 = Rs. 0.60

(i) EOQ = $\sqrt{\frac{2DC_s}{C_1}} = \sqrt{\frac{2 \times 600 \times 80}{0.60}}$ = 4000 units.

(ii) The minimum average yearly cost
= $Q^* x C_1$ = 4000x0.6 = Rs. 240

(iii) The optimum number of per year

$\frac{D}{q^*} = \frac{600}{400} = 1.5$

(iv) The optimum period of supply = $\frac{q^*}{D} = \frac{400}{600} = \frac{2}{3}$ year = 8 months.

(v) 200% of EOQ = 400 x $\frac{20}{100}$ = 80

∴ New ordering quality = 480 units

T.C. nCs + $\frac{1}{2}$ Q C_1

= $\frac{600}{400}$ x 80 + $\frac{1}{2}$ x 480 x 0.60

Increase is cent

= Rs. 244 – Rs. 240

= Rs. 4

Model III [Purchasing model with shortage]

Assumptions

(1) Annual demand 'D' is uniform at a rate of r units per unit time.
(2) Production is instantaneous.
(3) Zero lead time
(4) G is the inventory carrying cost per units per year.
(5) C_s is the setup cost
(6) C_2 is the shortage cost per unit/yr

Q = $\sqrt{\frac{2DCs}{C_1}\left(\frac{C_1+C_2}{C_2}\right)}$

Maximum inventory level = x $\left(\frac{C_2}{C_1+C_2}\right)$ and $t^* = \frac{q^*}{D}$

Minimum cost CA (ϕ) = $\sqrt{2DC_1Cs\ \frac{C_2}{C_1+C_2}}$