PROBABILITY AND STATISTICS

Q:1:-How many circular permutations are there of four persons playing bridge?

Solution

If we arbitrarily consider the position of one of the four players as fixed, we can seat

(arrange) the other three players in $3! = 6$ different ways. In other words, there are

six different circular permutations.

Q:2:-An assembler of electronic equipment has 20 integrated-circuit chips on her table, and she must solder three of them as part of a larger component. In how many ways can she choose the three chips for assembly?

Solution

Using Theorem 6, we obtain the result

$_{20}P_3 = 20!/17! = 20 \cdot 19 \cdot 18 = 6{,}840$

Q:3:-Describe a sample space that might be appropriate for an experiment in which we roll a pair of dice, one red and one green. (The different colors are used to emphasize that the dice are distinct from one another.)

Solution

The sample space that provides the most information consists of the 36 points given by

$S_1 = \{(x, y) | x = 1, 2, \ldots, 6; y = 1, 2, \ldots, 6\}$

where x represents the number turned up by the red die and y represents the number turned up by the green die. A second sample space, adequate for most purposes (though less desirable in general as it provides less information), is given by

$S_2 = \{2, 3, 4, \ldots, 12\}$

where the elements are the totals of the numbers turned up by the two dice

Q:4:-Near a certain exit of I-17, the probabilities are 0.23 and 0.24, respectively, that a truck stopped at a roadblock will have faulty brakes or badly worn tires. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and/or badly worn tires. What is the probability that a truck stopped at this roadblock will have faulty brakes as well as badly worn tires?

Solution

If B is the event that a truck stopped at the roadblock will have faulty brakes and T

is the event that it will have badly worn tires, we have $P(B) = 0.23$, $P(T) = 0.24$, and

$P(B \cup T) = 0.38$; substitution into the formula of Theorem 7 yields

$0.38 = 0.23 + 0.24 - P(B \cap T)$

Solving for $P(B \cap T)$, we thus get

$P(B \cap T) = 0.23 + 0.24 - 0.38 = 0.09$

Q:5:-If a person visits his dentist, suppose that the probability that he will have his teeth

cleaned is 0.44, the probability that he will have a cavity filled is 0.24, the probability that he will have a tooth extracted is 0.21, the probability that he will have his teeth cleaned and a cavity filled is 0.08, the

probability that he will have his teeth cleaned and a tooth extracted is 0.11, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist will have at least one of these things done to him?

Solution

If C is the event that the person will have his teeth cleaned, F is the event that he

will have a cavity filled, and E is the event that he will have a tooth extracted, we

are given $P(C) = 0.44$, $P(F) = 0.24$, $P(E) = 0.21$, $P(C \cap F) = 0.08$, $P(C \cap E) = 0.11$,

$P(F \cap E) = 0.07$, and $P(C \cap F \cap E) = 0.03$, and substitution into the formula of

Theorem 8 yields

$P(C \cup F \cup E) = 0.44 + 0.24 + 0.21 - 0.08 - 0.11 - 0.07 + 0.03$

$= 0.66$

If A and B are independent, then A and B are also independent.

Proof Since $A = (A \cap B) \cup (A \cap B)$, $A \cap B$ and $A \cap$
B

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are mutually exclusive, and A and

B are independent by assumption, we have

$P(A) = P[(A \cap B) \cup (A \cap B))]$

$= P(A \cap B) + P(A \cap B))$

$= P(A) \cdot P(B) + P(A \cap B))$

It follows that

$P(A \cap B) = P(A) - P(A) \cdot P(B)$

$= P(A) \cdot [1 - P(B)]$

$= P(A) \cdot P(B)$

and hence that A and B’ are independent

Q:6:-The completion of a construction job may be delayed because of a strike. The probabilities are 0.60 that there will be a strike, 0.85 that the construction job will be completed on time if there is no strike, and 0.35 that the construction job will be completed on time if there is a strike. What is the probability that the construction job will be completed on time?

Solution

If A is the event that the construction job will be completed on time and B is the event that there will be a strike, we are given $P(B) = 0.60$, $P(A|B) = 0.85$, and $P(A|B) = 0.35$. Making use of the formula of part (a) of Exercise 3, the fact that $A \cap B$ and $A \cap B'$ are mutually exclusive, and the alternative form of the multiplication rule, we can write

$P(A) = P[(A \cap B) \cup (A \cap B)]$

$= P(A \cap B) + P(A \cap B) = P(B) \cdot P(A|B) + P(B) \cdot P(A|B)$

Then, substituting the given numerical values, we get

$P(A) = (0.60)(0.35) + (1 - 0.60)(0.85)$

= 0.55

Q:7:- Determine the value of k for which the function given by

f(x, y) = kxy for x = 1, 2, 3; y = 1, 2, 3

can serve as a joint probability distribution.

Substituting the various values of x and y, we get f(1, 1) = k, f(1, 2) = 2k, f(1, 3) =

3k, f(2, 1) = 2k, f(2, 2) = 4k, f(2, 3) = 6k, f(3, 1) = 3k, f(3, 2) = 6k, and f(3, 3) = 9k.

To satisfy the first condition of Theorem 7, the constant k must be nonnegative, and

to satisfy the second condition,

k + 2k + 3k + 2k + 4k + 6k + 3k + 6k + 9k = 1

so that 36k = 1 and k = 1/36 .

Q:8:- If the random variables X, Y, and Z have the means μX = 3,μY = 5, and μZ = 2, the variances σ2,X = 8, σ2,Y = 12, and σ2,Z = 18, andcov(X, Y) = 1, cov(X, Z) = −3,and cov(Y, Z) = 2, find the covariance ofU = X + 4Y + 2Z and V = 3X − Y − Z

Solution

By Theorem 15, we get

cov(U, V) = cov(X + 4Y + 2Z, 3X − Y − Z)

= 3 var(X) − 4 var(Y) − 2 var(Z) + 11 cov(X, Y)

+ 5 cov(X, Z) − 6 cov(Y, Z)

= 3 · 8 − 4 · 12 − 2 · 18 + 11 · 1 + 5(−3) − 6 · 2

= −76

Q:9:-If the probability is 0.75 that an applicant for a driver's license will pass the road test

on any given try, what is the probability that an applicant will finally pass the test on

the fourth try?

Solution

Substituting x = 4 and θ = 0.75 into the formula for the geometric distribution,we get

$g(4; 0.75) = 0.75(1 - 0.75)^{4-1}$

$= 0.75(0.25)^3$

= 0.0117

Of course, this result is based on the assumption that the trials are all independent,

and there may be some question here about its validity

Q:10:-The average number of trucks arriving on any one day at a truck depot in a certain

city is known to be 12. What is the probability that on a given day fewer than 9 trucks

will arrive at this depot?

Solution

Let X be the number of trucks arriving on a given day. Then, using Table II of "Statistical Tables" with λ = 12, we get

$$P(X < 9) = \sum_{x=0}^{8} p(x; 12) = 0.1550$$

11Which of the following statements is not correct concerning the probability distribution of a continuous random variable?

a. the vertical coordinate is the probability density function

b. the range of the random variable is found on the x-axis

c. the total area represented under the curve will equal 1

d. the area under the curve between points a and b represents the probability that X = a

e. the area under the curve represents the sum of probabilities for all possible outcomes

Answer: D

12. Which of the following is not a characteristic of the normal distribution?

a. it is a symmetrical distribution

b. the mean is always zero

c. the mean, median and mode are equal

d. it is a bell-shaped distribution

e. the area under the curve equals one

Answer: B

13. Which of the following is not a correct statement?

a. the exponential distribution describes the Poisson process as a continuous random variable

b. the exponential distribution is a family of curves, which are completely described by the mean

c. the mean of the exponential distribution is the inverse of the mean of the Poisson

d. the Poisson is a probability distribution for a discrete random variable while the exponential distribution is continuous

e. the area under the curve for an exponential distribution equals 1

Answer: C

14. Which of the following do the normal distribution and the exponential density function have in common?

a. both are bell-shaped

b. both are symmetrical distributions

c. both approach infinity as x approaches infinity

d. both approach zero as x approaches infinity

e. all of the above are features common to both distributions

Answer: D

15. Which of the following statement is not true for an exponential distribution with parameter λ?

a. mean = 1 / λ

b. standard deviation = 1 / λ

c. the distribution is completely determined once the value of λ is known

d. the area under the curve is equal to one

e. the distribution is a two-parameter distribution since the mean and standard deviation are equal

Answer: E

16. Which of the following distributions is suitable to model the length of time that elapses before the first employee passes through the security door of a company?

a. exponential

b. normal

c. poisson

d. binomial

e. uniform

Answer: A

17. Which of the following distributions is suitable to measure the length of time that elapses between the arrival of cars at a petrol station pump?

a. normal

b. binomial

c. uniform

d. poisson

e. exponential

Answer: E

18. A multiple-choice test has 30 questions. There are 4 choices for each question. A student who has not studied for the test decides to answer all the questions randomly by guessing the answer to each question. Which of the following probability distributions can be used to calculate the student's chance of getting at least 20 questions right?

a. Binomial distribution

b. Poisson distribution

c. Exponential distribution

d. Uniform distribution

e. Normal distribution

Answer: A

19. It is known that 20 % of all vehicles parked on campus during the week do not have the required parking disk. A random sample of 10 cars is observed one Monday morning and X is the number in the sample that do not have the required parking disk. We can assume here that the probability distribution of X is:

a. Binomial

b. Normal

c. Poisson

d. Exponential

e. Any continuous distribution will do

Answer: A

20. Which of the following statements is/are true regarding the normal distribution curve?

a. it is symmetrical

b. it is bell-shaped

c. it is asymptotic in that each end approaches the horizontal axis but never reaches it

d. its mean, median and mode are located at the same point

e. all of the above statements are true

Answer: E

21. In a popular shopping centre, the waiting time for an ABSA ATM machine is found to be uniformly distributed between 1 and 5 minutes. What is the probability of waiting between 2 and 3 minutes to use the ATM?

a. 0.25

b. 0.50

c. 0.75

d. 0.20

e. 0.40

Answer: A

22. In a popular shopping centre, the waiting time for an ABSA ATM machine is found to be uniformly distributed between 1 and 5 minutes. What is the probability of waiting between 2 and 4 minutes to use the ATM?

a. 0.25

b. 0.50

c. 0.75

d. 0.20

e. 0.40

Answer: B