

Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32357504
Name of Paper	: DSE-II Mathematical Finance
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

- Determine the effective rates of interest corresponding to the 5% rate of interest compounded daily, weekly, monthly, quarterly, semi-annually, and continuously. Arrange them in increasing order.
- If one-year and two-year spot rates are $s_1 = 4\%$ and $s_2 = 5\%$ respectively, then find the forward rate f_{12} . If the spot rate curve is (4.0, 5.0, 5.4, 5.6, 5.8, 6.0), then find the spot rate curve of the next year.
- Assume that there are three assets having mean rates of return $\bar{r}_1 = 8\%$, $\bar{r}_2 = 10\%$, $\bar{r}_3 = 6\%$, standard deviations $\sigma_1 = 1.5$, $\sigma_2 = 0.5$, $\sigma_3 = 1.2$ and correlations $\rho_{12} = 0.3$, $\rho_{23} = 0$, $\rho_{13} = -0.2$.
 - Find the covariance matrix for these three assets.
 - Find the minimum-variance portfolio.
 - Find another efficient portfolio by setting $\lambda = 1$, $\mu = 0$.
 - If the risk-free rate is $r_f = 5\%$, then find the one fund of risky assets as specified in one-fund theorem.
- Consider a market in which there are only two risky assets A and B , and a risk-free asset F . The total values of the assets in the market are \$200,000 and \$100,000, respectively. The market portfolio M consists of assets A and B in proportion to their market values. The following information is known: $r_F = .07$, $\bar{r}_A = .12$, $\bar{r}_B = .15$, $\sigma_A^2 = .06$, $\sigma_B^2 = .09$, $\sigma_{AB} = .01$.
 - Write down the expression for the market portfolio in terms of the weights w_1 and w_2 of the two assets A and B in the market portfolio.
 - Find the expected return and the standard deviation of the market portfolio M .
 - Calculate σ_{AM} , σ_{BM} and hence find the beta of the stocks A and B .
 - Verify that the beta of the market portfolio M is 1.
- Consider a long forward contract to purchase a non-dividend paying stock in 6 months. The current price of the stock is \$100 and the six-month risk-free rate is 10% per annum compounding continuously. Obtain the forward price and discuss the arbitrage opportunities if the forward price is \$104.

6. The price of a non-dividend paying stock is \$19 and the price of a three-month European call option on the above stock with strike price \$20 is \$1. The risk-free rate is 4% per annum compounding continuously. Find the price of a 3-month European put option written on the same stock with strike price \$20? If the above European put option is selling at \$ 2, then identify an arbitrage opportunity if it exists.