(This Question paper contains 4 printed pages)

Roll No..........

Sr. No. of Question Paper: 8904

Unique Paper Code :235505

Name of the Course :BSc(Hons) Mathematics

Name of the Paper : Linear Programming and Theory of Games

Semester : V

J

Duration: 3 Hours

Maximum Marks: 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts of each question.
3. All questions carry equal marks.

1.(a) If for a basic feasible solution $x_B = B^{-1}b$ to the linear programming problem:

Minimize $z = cx$

$Ax = b.$ Insert

$x \geq 0$

there is some column x_j in A but not in B for which $z_j - c_j > 0$ and $y_{ij} \leq 0$, then prove that the problem has an unbounded solution.

(b) Let $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ is be a feasible solution of the system of equations

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

Reduce this feasible solution to a basic feasible solution.

(c) Solve the following LPP using Simplex method:

Max $z = 10x_1 + x_2 + 2x_3$

Subject to

$$x_1 + x_2 - 2x_3 \leq 10$$

$$4x_1 + x_2 + x_3 \leq 20$$

$x_1, x_2 \geq 0$, x_3 unrestricted.

2) (a) Solve the linear programming problem by Big M method.

$$\text{Max } z = 3x_1 - x_2$$

Subject to

$$2x_1 + x_2 \geq 2$$

$$x_1 + 3x_2 \leq 3$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

(b) Use Simplex method to solve the system of equation:

$$3x_1 + 2x_2 = 5$$

$$2x_1 + x_2 = 4$$

(c) Find the dual of the following LPP

Minimize $Z = x_1 + x_2 + 2x_3$

subject to $x_1 + x_2 + x_3 \leq 9$

$$2x_1 - 3x_2 + 3x_3 \geq 1$$

$$-3x_1 + 6x_2 - 4x_3 = 3$$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ unrestricted.

3 (a) Solve the following LPP by Two Phase method.

$$\text{Max } z = x_1 + 5x_2$$

Subject to

$$3x_1 + 4x_2 \leq 6$$

$$x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0.$$

(b) State and prove Strong Duality theorem.

(c) Apply the principle of duality to solve the linear programming problem

Minimize $x = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \leq 10$$

$$x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

4) (a) Solve the following cost-minimizing transportation problem:

	D_1	D_2	D_3	Supply
O_1	6	8	4	14
O_2	4	9	3	12
O_3	6	10	15	5
Demand	6	10	15	

(b) Minimize the total man hours in the following assignment matrix by Hungarian method.

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

(c) Define saddle point of a two person zero sum game. Use the minimax criteria to find the best strategy for each player for the game having the following pay off matrix

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{bmatrix} 1 & -1 \\ -2 & 0 \\ 3 & 1 \end{bmatrix} \end{array}$$

Is it a stable game?

5) (a) Solve graphically the game whose payoff matrix is

$$\begin{bmatrix} 2 & 4 & 11 \\ 7 & 4 & 2 \end{bmatrix}$$

(b) Use the relation of dominance to solve the game whose payoff matrix is given by

$$\begin{bmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

(c) Reduce the following game to a Linear Programming Problem and then solve by simplex method.

Shift it to the previous sheet.

$$\begin{bmatrix} 1 & -3 & 2 \\ -4 & 4 & -2 \end{bmatrix}$$