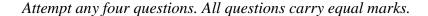
Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32351601
Name of Paper	: C 13- Complex Analysis
Semester	: VI
Duration	: 3 hours
Maximum Marks	: 75 Marks



1. Determine whether $S = \{z \in \mathbb{C} : |z|^2 > z + \overline{z}\}$ is a domain or not? Justify your answer.

Find the image of line segment joining $z_1 = -i$ to $z_2 = -1$ under the map $f(z) = \overline{iz}$.

Check whether Cauchy-Riemann equations for $f(z) = \sqrt{|z^2 - \overline{z}^2|}$ are satisfied at the origin? Is f analytic at the origin? Justify your answer.

Suppose $f(z) = \cosh(2x)\cos(2y) + iv(x, y)$ is analytic everywhere such that v(0,0) = 0. Find f(z). Hence find zeros of f.

Solve the equation $e^{z-1} + ie^3 = 0$.

2. Let $S = \{z \in \mathbb{C} : \text{Im } z = 1 \text{ and } \text{Re } z \neq 4\}$. Is S open? Is S closed? Justify your answer.

Assume that g is analytic in a region and that at every point either g = 0 or g' = 0. Show that g is constant.

Suppose $f(z) = \begin{cases} \overline{z}^3/z^2 & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Show that *f* is continuous everywhere on \mathbb{C} . Is *f* analytic at z = 0? Justify your answer.

Does there exists an analytic function f(z) = u(x, y) + i v(x, y) for which $u(x, y) = y^3 + 5x$? Solve the equation $Log(z) + Log(2z) = 3\pi/2$.

3. Determine whether the following curves are simple, closed, smooth or contour

 $\begin{array}{ll} C_1: \ z(t) = |t| + it \ , & t \in [-1,1] \\ C_2: \ z(t) = \ e^{2it} \ , & t \in [0, \ 2\pi] \ , \\ C_3: z(t) \ \text{is the positively oriented boundary of the rectangle whose sides lie along} \\ & x = \pm 1, \ y = 0, \ y = 1. \end{array}$

Evaluate $\int_{C_3} |z| dz$. Explain why Cauchy Goursat theorem is not applicable in this case? Use ML-Inequality to show that

$$\left| \int_C \frac{e^z}{(z+1)} dz \right| \le 4\pi e^2$$

where $C: z(t) = e^{2it}$, $t \in [-\pi, \pi]$.

4. Evaluate $\int_C ze^{3z} dz$ where *C* is the parabola $x^2 = y$ from (0,0) to (1,1). Using Cauchy Integral formula, determine the integral $\int_C \frac{e^z}{z^2(z^2-9)} dz$ where *C* is positively oriented circle (i) *C*: |z| = 1. (ii) *C*: |z - 3| = 1.

Use Liouville's theorem to establish that *cos z* is not bounded in the complex plane.

Let g be an entire function and suppose that |g(z)| < 10 for all values of z on the circle |z - 2| = 3. Find a bound for |g'''(2)|.

- 5. Determine the radius of convergence of the series ∑_{k=0}[∞] z^k/k! and ∑_{k=0}[∞] k^kz^k. Also discuss the convergence of the series.
 Obtain the Maclaurin series of the function (z) = 1/z² sinh (1/z). Specify the region in which the series is valid.
 Find the Laurent series of the function f(z) = 1/((z+1)(z+3)) valid for 0 < |z+1| < 2.
- 6. Determine the residue and singularities of the function $g(z) = \frac{z+1}{z^2+4}$. Also evaluate $\int_C g(z)dz$ where C is the positively oriented circle |z i| = 2. Using a single residue, evaluate the integral $\int_{C'} \frac{3z-1}{z(z+1)} dz$ where C' is the positively oriented circle |z - 1| = 4.

Use residue to evaluate the integral $\int_0^{2\pi} \frac{dt}{3+cost}$