Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8081**

Unique Paper Code : **32357501** **J**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (*a*) A real root of the equation $x^3 - 5x + 1 = 0$ lies in $]0, 1[$. Perform three iterations of Regula Falsi Method to obtain the root. 6

(b) Perform three iteration of Bisection Method to obtain root of the equation $\cos(x) - xe^x$ in $]0, 1[$. 6

(c) Discuss the order of convergence of the Secant method and give the geometrical interpretation of the method. 6

2. (a) Verify $x = \sqrt{a}$ is a fixed point of the function $h(x) = \frac{1}{2}\left(x + \frac{a}{x^2}\right)$. Determine order of convergence of sequence $p_n = h(p_{n-1})$ towards $x = \sqrt{a}$. 6.5

(b) Use Secant method to find root of $3x + \sin(x) - e^x = 0$ in $]0, 1[$. Perform three iterations. 6.5

(c) Prove that Newton's Method is of order two using $x^3 + 2x^2 - 3x - 1 = 0$ and initial approximation $x_0 = 2$. 6.5

3. (a) Define a lower and an upper triangular matrix. Solve the system of equations :

$$-3x_1 + 2x_2 - x_3 = -12$$

$$6x_1 + 8x_2 + x_3 = 1$$

$$4x_1 + 2x_2 + 7x_3 = 1$$

by obtaining an LU decomposition of the coefficient matrix A of the above system. 6

(*b*) For Jacobi method, calculate T_{jac}, C_{jac} and spectral radius of the following matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$ 6

(*c*) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$4x_1 + 2x_2 - x_3 = 1$$

$$2x_1 + 4x_2 + x_3 = -1$$

$$-x_1 + x_2 + 4x_3 = 1$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6

4. (*a*) Construct the Lagrange form of the interpolating polynomial from the following data :

x	1	2	3
$f(x) = \ln x$	ln 1	ln 2	ln 3

6.5

(b) Prove that for $n + 1$ distinct nodal points $x_0, x_1, x_2, \ldots x_n$, there exists a unique interpolating polynomial of at most degree n. 6.5

(c) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval [0, 1] so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}. 6.5

5. (a) Define the backward difference operator ∇ and the Newton divided difference. Prove that :

$$f[x_0, x_1, -----, x_n] = \frac{\nabla^n f_n}{n!h^n} \text{ where } h = x_{i+1} - x_i. \qquad 6$$

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	–7	–5	–4	–1
y	10	5	2	10

Find the approximation of y for $x = -3$. 6

(c) Use the formula

$$f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}$$

to approximate the derivative of $f(x)=1+x+x^3$ at $x_0 = 1$ taking $h = 1, 0.1, 0.001$. What is the order of approximation ? 6

6. (a) Approximate the value of $\int_0^1 e^{-x}dx$ using the Trapezoidal rule and verify that the theoretical error bound holds for the same. 6.5

(b) State Simpson's 1/3rd rule for the evaluation of $\int_a^b f(x)dx$ and prove that it has degree of precision 3. 6.5

(c) Use Euler's method to approximate the solution of the initial value problem.

$x' = (1+x^2)/t$, $x(1)=0$, $1 \le t \le 4$ taking 5 steps. 6.5