This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **7945**

Unique Paper Code : **32357505** **J**

Name of the Paper : **Discrete Mathematics**

Name of the Course : **B.Sc. (Hons.) Mathematics : DSE-1**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *two* parts from each question.

SECTION I

1. (*a*) (*i*) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as :

For $m, n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \in N_0 : n = km$, then show that $\leq$ is an order relation on N_0.

(*ii*) Draw Hasse diagram for the subset $P = \{1, 2, 3, 12, 18, 0\}$ of $(N_0; \leq)$, where $\leq$ same as defined above. 3+3 (*b*) Show that two finite ordered sets P and Q are order isomorphic iff they can be drawn with identical diagrams. 6

(*c*) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi : P \to Q$ and $\psi : Q \to P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \to S$ denotes the identity map on S given by : $id_S(x) = x$, $\forall\, x \in S$. 6

2. (*a*) Let $(L, \wedge, \vee)$ be a non-empty set equipped with two binary operations $\wedge$ and $\vee$. Also L is such that the following laws, associative law, commutative law, idempotency law and absorption law and their duals hold. Then show that :

(*i*) $(a \vee b) = b$ iff $(a \wedge b) = a (\forall\, a, b \in L)$

(*ii*) Define a relation $\leq$ on L as $a \leq b$ if $(a \vee b) = b$. Then prove that $\leq$ is an order relation on L. 6.5

(b) Let L and K be lattices and $f : L \to K$ be a map. Then show that the following are equivalent :

(i) f is order preserving

(ii) $(\forall\ a, b, \in L)\ f(a \vee b) \geq f(a) \vee f(b)$. 6.5

(c) Prove that in any lattice L, we have :

$$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$$

$(\forall\ x, y, z \in L)$. 6.5

SECTION II

(a) Let L be a lattice. Prove that L is distributive if and only if for all elements a, b, c of L,

$(a \vee b = c \vee b$ and $a \wedge b = c \wedge b)$ implies $a = c$. 6

(b) Find the conjunctive normal form of $f = (x(y' + z)) + z'$ in three variables. Also find its disjunctive normal form. 6

(c) Prove that every Boolean algebra is sectionally complemented. 6

4. (*a*) Find the prime implicants of $xy + xy'z + x'y'z$ and form the corresponding prime implicant table. 6.5

(*b*) Simplify the following function using the Karnaugh diagram : 6.5

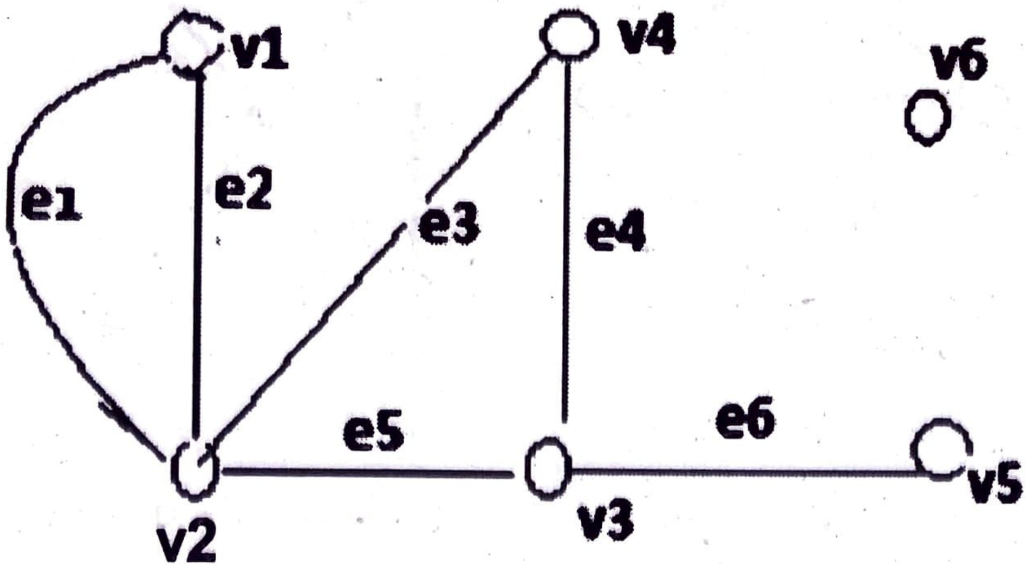
$$x_1x_2x'_3 + x'_1x_2x'_3 + (x_1 + x'_2x'_3)(x_1 + x_2 + x_3)' + x_3(x'_1 + x_2).$$

(*c*) A motor is supplied by three generators. The operation of each generator is monitored by a corresponding switching element which closes a circuit as soon as generator fails. In the electrical monitoring system, a warning lamp lights up if one or two generators fail. Determine a symbolic representation as a mathematical model of this problem. 6.5

SECTION III

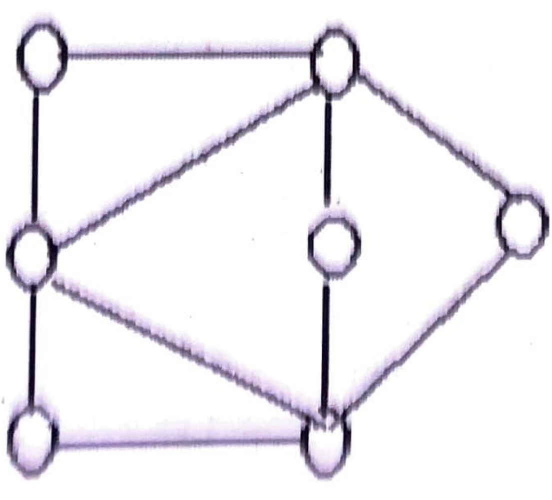
5. (*a*) (*i*) Prove that number of odd vertices in a pseudo graph is even.

(*ii*) Find the degree sequence for G; verify that the sum of the degrees of the vertices is an even number. Which vertices are even ? Which are odd ?

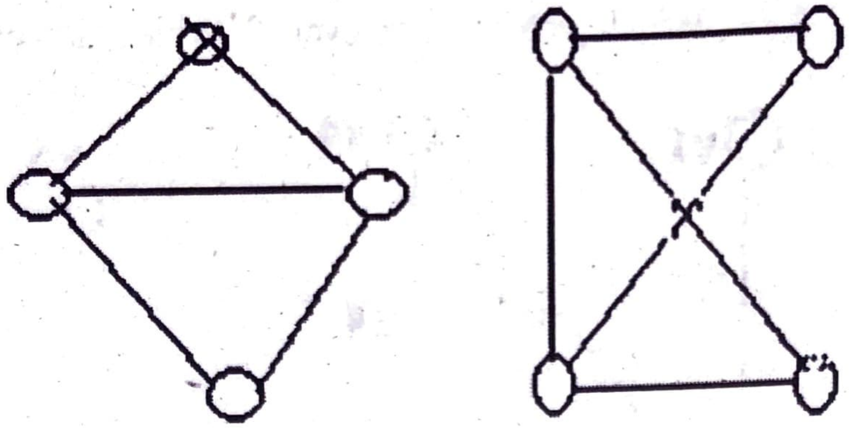


2+4

(*b*) (*i*) What is bipartite graph ? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.



(*ii*) Define isomorphism of graph. Also label the graphs so as to show an isomorphism.

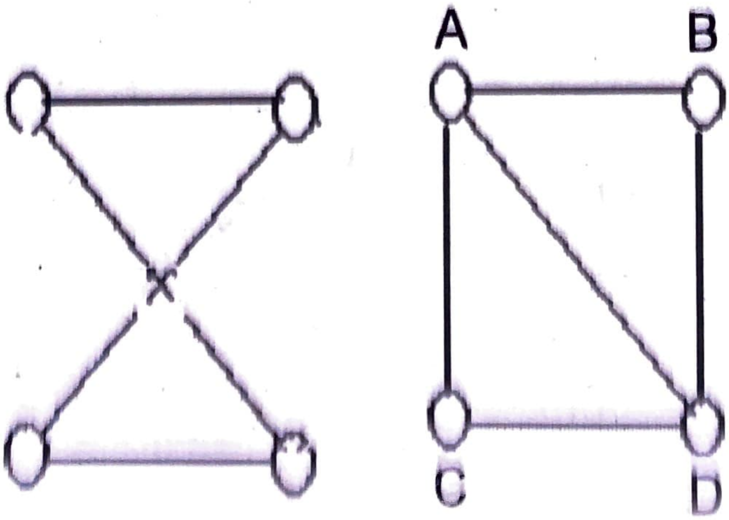


3+3

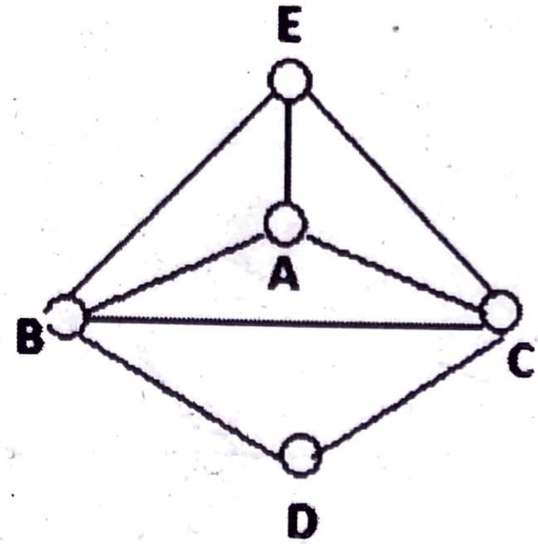
(*c*) (*i*) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have ?

(*ii*) Why can there not exist a graph whose degree sequence is 5, 4, 4, 3, 2, 1.

(*iii*) Explain why the graphs are not isomorphic.

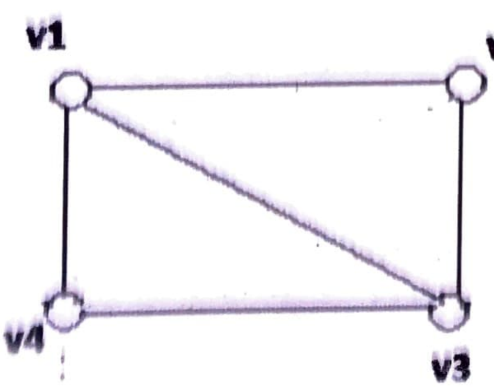


6. (a) (i) Define Hamiltonian graph. Is the graph given below Hamiltonian ? If no, explain. If yes, find a Hamiltonian cycle.

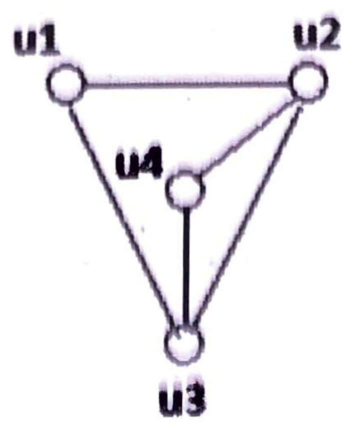


(ii) Answer the Konisberg bridge problem and explain. 6.5

(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 as shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.



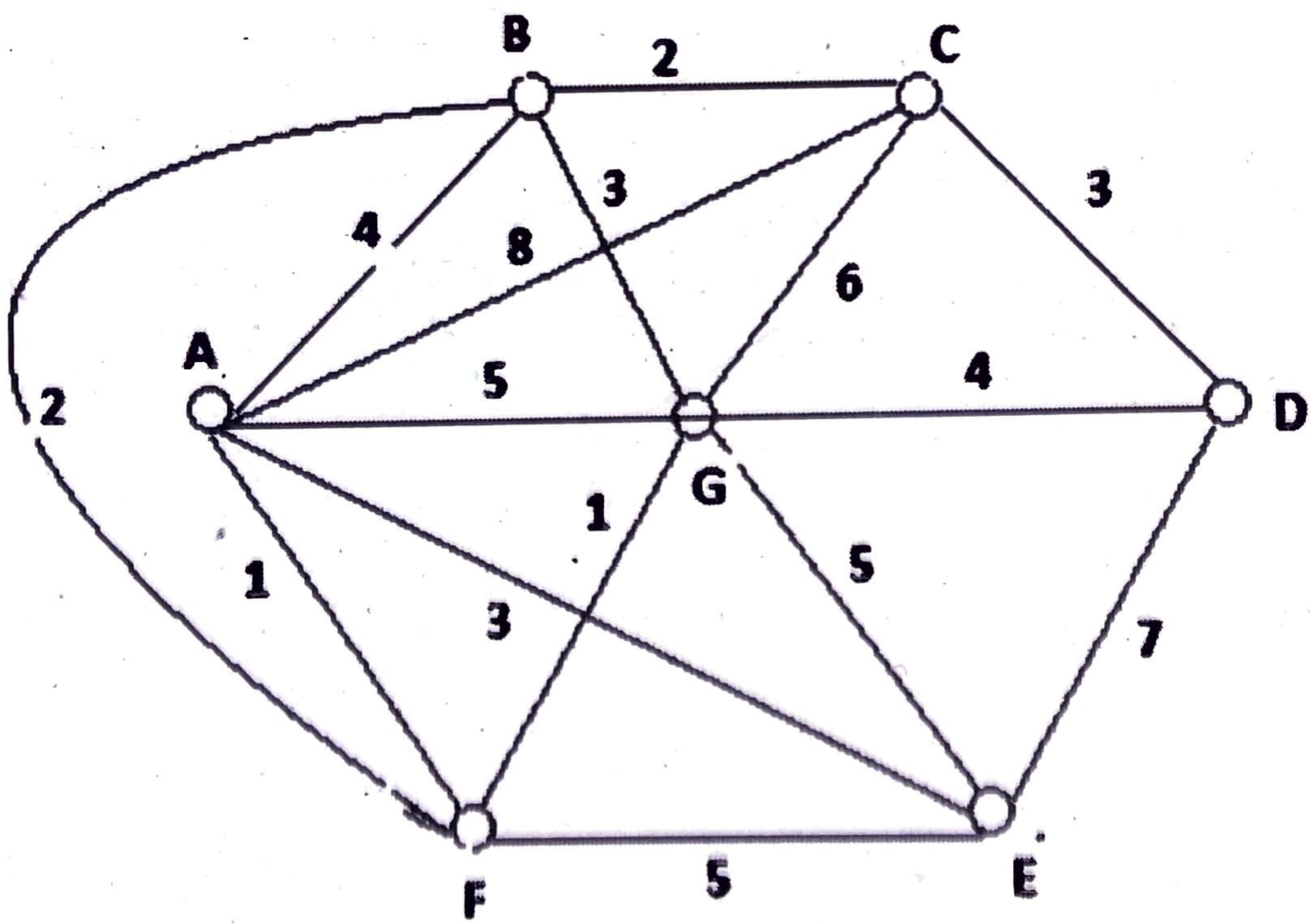
G_1



G_2

6.5

(c) Apply the improved version of Dijkstra's algorithm to find the length of a shortest path from A to D in the graph shown below. Write steps.



6.5