[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **2814** GC-4

Unique Paper Code : 32351401

Name of the Paper : C-8 Partial Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics – II**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Marks of each part are indicated.

SECTION – I

1. Attempt any **two** parts out of the following :

(a) Find the solution of the Cauchy problem

$$u_x + xu_y = \left(y - \frac{1}{2}x^2\right)^2, \text{ with } u(0, y) = \exp(y). \qquad (7\tfrac{1}{2})$$

P.T.O.

(b) Using $v = \ln u$ and $v = f(x) + g(y)$, find the solution of the Cauchy problem

$$y^2u_x^2 + x^2u_y^2 = (xyu)^2, \quad \text{with } u(x,0) = e^{x^2}.$$ (7½)

(c) Find the integral surfaces of the equation $uu_x + u_y = 1$ for the following initial data : $x(s,0) = s$, $y(s,0) = 2s$, $u(s,0) = s$. (7½)

SECTION – II

2. Attempt any **one** part out of the following :

(a) State Conservation Law and derive the Burger's Equation

$$u_t + uu_x = \nu u_{xx},$$

where ν is the kinematic viscosity and $u(x,t)$ is the fluid velocity field. (6)

(b) Derive the damped wave equation of a string

$$u_{tt} + au_t = c^2u_{xx},$$

where the damping force is proportional to the velocity and a is constant. Considering a restoring force proportional to the displacement of a string, show that the resulting equation is

$u_{tt} + au_t + bu = c^2u_{xx}$, where b is a constant. (6)

3. Attempt any **two** parts out of the following :

(a) Find the characteristics, characteristic coordinates and reduce the equation given below to the canonical form : elliptic

$B^2 - 4AC < 0$

$$u_{xx} + x^2u_{yy} = 0, \quad \text{for } x \neq 0.$$ (6)

(b) Use the polar coordinates r and θ ($x = r\cos\theta$, $y = r\sin\theta$) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into the polar form

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$ (6)

(c) Obtain the general solution of the equation given below :

$$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} + xyu_x + y^2u_y = 0$$ (6)

$u(x,y) = f(y/x)e^{-y} + g(y/x)$ Parabolic

SECTION – III

4. Attempt any **three** parts out of the following :

(a) Determine the solution of the Cauchy problem given below :

$$u_{tt} - 9u_{xx} = 0, \ x \in R, \ t > 0$$

$$u(x,0) = \cos x, \qquad u_t(x,0) = \sin 2x$$ (7)

P.T.O.

$u(x,t) = \cos x \cos 3t + \frac{1}{6}\sin 2x \sin 6t$

(b) Find the solution of the initial boundary- value problem

$$u_{tt} = 4u_{xx}, \quad 0 < x < 1, \quad t > 0,$$

$$u(x,0) = 0, \quad 0 \le x \le 1,$$

$$u_t(x,0) = x(1-x), \quad 0 \le x \le 1,$$

$$u(0,t) = 0, \; u(1,t) = 0, \quad t \ge 0.$$

(7)

(c) Solve the Cauchy problem for the non- homogeneous wave equation

$$u_{tt} = c^2 u_{xx} + h^*(x,t),$$

with the initial conditions

$$u(x,0) = f(x), \quad u_t(x,0) = g^*(x)$$

(7)

(d) Solve the characteristic initial- value problem

$$xu_{xx} - x^3 u_{yy} - u_x = 0, \; x \ne 0,$$

$$u(x,y) = f(y) \text{ on } y - \frac{x^2}{2} = 0 \text{ for } 0 \le y \le 2,$$

$$u(x,y) = g(y) \text{ on } y + \frac{x^2}{2} = 4 \text{ for } 2 \le y \le 4,$$

where $f(2) = g(2)$.

(7)

SECTION – IV

5. Attempt any **three** parts out of the following :

(a) Discuss the solution of the Heat Conduction Problem

$$u_t = ku_{xx}, \; 0 < x < l, \; t > 0$$

$$u(0,t) = 0, \; t \ge 0$$

$$u(l,t) = 0, \; t \ge 0$$

$$u(x,0) = f(x), \; 0 \le x \le 1.$$

(7)

(b) Solve using the method of separation of variables

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < \pi, \quad t > 0,$$

$$u(x,0) = \sin x, \quad u_t(x,0) = x^2 - \pi x, \quad 0 \le x \le \pi,$$

$$u(0,t) = u(\pi,t) = 0, \quad t > 0.$$

(7)

(c) Determine the solution of

$$u_{tt} = c^2 u_{xx} + A \sinh x, \; 0 < x < l, \quad t > 0,$$

$$u,(x,0) = 0, \quad u_t(x,0) = 0, \quad 0 \le x \le l,$$

$$u(0,t) = h, \quad u(l,t) = k, \quad t > 0$$

where h, k and A are constants.

(7)

(d) Find the temperature distribution in a rod of length l. The faces are insulated, and the initial temperature distribution is given by $x(l-x)$. (7)