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[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7465** **J**

Unique Paper Code : 32351303

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Multivariate Calculus

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** Sections are compulsory.

(iii) Attempt any **five** questions from each **Section**.

(iv) All questions carry equal marks.

P.T.O.

Section- I

1. Given that the function

$$f(x,y)=\begin{cases}\dfrac{3x^3-3y^3}{x^2-y^2} & \text{for } x^2 \neq y^2\\ B & \text{otherwise}\end{cases}$$

is continuous at the origin, what is B ?

2. In physics, the *wave equation* is :

$$\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is :

$$\frac{\partial z}{\partial t}=c^2\frac{\partial^2 z}{\partial x^2}$$

Determine whether $z = \sin 5ct \cos 5x$ satisfies the wave equation, the heat equation, or neither.

3. The radius and height of a right circular cone are measured with errors of at most 3% and 2%, respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.

4. If $f(x, y, z) = xy^2e^{xz}$ and $x = 2 + 3t$, $y = 6 - 4t$, $z = t^2$. Compute $\frac{df}{dt}(1)$.

5. Sketch the level curve corresponding to $C = 1$ for the function $f(x,y) = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ and find a unit normal vector at the point $P_0\left(2\sqrt{3}\right)$.

6. Find the point on the plane $2x + y - z = 5$ that is closest to the origin.

Section - II

7. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \le 1$ in the first quadrant.

8. Sketch the region of integration and then compute the integral $\int_0^1 \int_x^{2x} e^{y-x}\, dy\, dx$ in 2 ways :

   (a) with the given order of integration

   (b) with the order of integration reversed

9. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.

10. Find the volume of the tetrahedron bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.

11. Compute $\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

12. Use the change of variables to compute $\iint_D \frac{(x-y)^4}{(x+y)^4}\,dydx$, where D is the triangular region bounded by the line $x + y = 1$ and the coordinate axes.

Section - III

13. Find the work done by the force field $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\vec{i} - \frac{y}{\sqrt{x^2+y^2}}\vec{j}$ when an object moves from (a,0) to (0,a) on the path $x^2+y^2=a^2$.

14. Verify that the following line integral is independent of the path $\oint(3x^2+2x+y^2)$ $dx + (2xy + y^3)\,dy$ where C is any path from (0,0) to (0,1).

15. Use Green's theorem to evaluate $\oint_C \left(x \sin x\,dx - \exp\left(y^2\right)dy\right)$ where C is the closed curve joining the points (1,-1) (2,5) and (-1,-1) in counterclockwise direction.

16. State Stoke's theorem and use it to evaluate $\iint_S \text{curl}\bar{F}.d\bar{S}$ where $\bar{F} = xz\bar{i} + yz\bar{j} + xy\bar{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

17. Use the divergence theorem to evaluate the surface integral $\iint_S \bar{F}.\bar{N}dS$, where $\bar{F} = \left(x^2 + y^2 - z^2\right)\bar{i} + yx^2\bar{j} + 3z\bar{k}$; S is the surface comprised of the five faces of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, missing $z = 0$.

18. Evaluate $\iint_S 2x\,dS$ where S is the portion of the plane $x + y + z = 1$ with $x \geq 0,\ y \geq 0,\ z \geq 0$.