Your Roll No.............

Sr. No. of Question Paper: 6032

Unique Paper Code   : 12481202

Name of the paper   : Mathematics for Business
                            Economics

Name of the Course   : **B.A. (Hons.) Business
                            Economics, 2022 (CBCS)**

Semester   : II

Duration   : 3 Hours

Maximum Marks   : 75

Instructions for Candidates

1.   Write your Roll No. on the top immediately on receipt
     of this question paper.

2.   Attempt all questions.

3.   Choice is available within each question. Use of simple
     calculator is allowed.

1.   Attempt any three questions: (5×3=15)

(a) (i) Solve $|x+5| \leq 2$

(ii) Determine the value of k so that the function is
     continuous at $x = 1$

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$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 1}, & x \neq 0 \\ k, & x = 1 \end{cases}$$

(b) (i) Define rank of a matrix. Find the value of x such that A is a less than full rank matrix,

$$\text{Where } A = \begin{pmatrix} 3 & 5 & 0 \\ 3 & x & 2 \\ 9 & -1 & 8 \end{pmatrix}.$$

(ii) Consider the function f defined by the formula f (x) = 4 In $\sqrt{x + 4} - 2$. For which values of x is f(x) defined? Determine the range of f.

(c) Calculate the limit for the two functions:

(i) $\displaystyle \lim_{x \to -\infty} \sqrt{\dfrac{2 + 3x}{x - 1}}$ (ii) $\displaystyle \lim_{x \to 0} \dfrac{x + |x|}{x}$

(d) (i) Define idempotence of a matrix. If $A = \begin{pmatrix} 2 & x \\ 3 & y \end{pmatrix}$ is an idempotent matrix, find x & y.

(ii) If a company spends Rs 4000 per month on advertisement, it sells 15,000 items per month. If it spends Rs 6,000 on advertisement, it sells 20,000 items per month. Assuming a linear relation between advertisement cost and number of items sold, what would be the number of items sold if it spends Rs 10,000 on advertisement.

2. Attempt any three questions: (5 x 3 = 15)

(a) (i) Find all integer roots of the following equations: $-2x^3 + 2x^2 + 10x + 6 = 0$

(ii) A machine costs Rs. 10,000 and provides a return of Rs. 3,000 after one year, Rs. 4,000 after two years and Rs. 500 from the third year till eternity. Check whether the purchase of the machine is profitable, assuming market rate of interest is 10%.

(b) State the conditions required for a system of m linear equations in n variables to have a unique solution, infinite solutions, and no solution. Use these conditions on the equations:

$$2x + 3y = m$$

$$6x + ky = 45$$

to find the values of k and m so that there exits:
(i) a unique solution, (ii) infinite solutions (ii) no solution.

(c) Given the following transaction table for an economy:

	Sector of destination		
Sector of Origin	1	2	3
1	10	20	60
2	40	40	90
3	30	80	100

The final demand for the three industries 1, 2. and 3 are given are given as 10, 30 and 130 respectively. (i) Write down the input coefficients matrix. (ii) Find the equilibrium output if the final demand changes to 50, 100, and 200 respectively. (iii) Write down the new transaction table.

(d) The growing value of fish in a pond (V) is given

by $V = 1000\, e\, \sqrt{0.5t}$ where t is the time in years. The annual rate of interest is 8 percent per annum. What is the optimum time to sell the fish?

3. Attempt any four questions:　　　　　$(5 \times 4 = 20)$

(a) (i) Suppose the function f is defined for all x by the following: $f(x) = x^5 + 3x^5 + 6x - 3$. Show that f has an inverse function g and find g '(7).

(ii) Find the linear approximation to
$f(x) = (5x + 3)^{-2}$ about $x = 0$.

(b) What is meant by saying that a set of points S in the plane is convex? Graph the following sets and indicate whether it is convex?

$$A = \left\{ (X,Y) : Y \geq \frac{2X+5}{X+2}, X>0 \right\}$$

$$B = \left\{ (X,Y) : Y \leq X^2 \right\}$$

(i) $C = A \cap B$ (ii) $C = A \cap B$

(c) A monopolist has a total revenue function given by $TR = 80Q-8Q^2$ and a total cost function given by $TC = 3Q^2 -8Q+50$.

(i) Find the level of output, Q, which maximises profit of the producer.

P.T.O.

(ii) What is the maximum value of tax rate t imposed on the monopolist that maximises tax revenue to the government?

(d) Find out the whether the following function is convex/concave and determine possible inflection points: $f(x) = xe^x$

(e) Let a, b, and c be the three linearly independent three-dimensional vectors and let

α = a - 2b; β = a + b + c; μ = b - c. Do α, β and μ also form a linearly independent set of vectors?

4.   Attempt any five questions: $(5 \times 5 = 25)$

(a) Show that the isoquants of the production function given below are well behaved:

$$x = \left[k^{\frac{1}{2}} + L^{\frac{1}{2}} \right]^2$$

(b) The demands for a monopolist's two products are determined by the equations p = 25 – x, q=24 – 2y where p and q are prices per unit of the two goods, and x and y are the corresponding quantities. The costs of producing and selling x

units of the first good and y units of the other are $C(x, y) = 3x^2 + 3xy + y^2$. Find the monopolist's maximum profit $\pi(x, y)$ from producing and selling x units of the first good and y units of the other.

(c) In the following case, find dz/dt by using the chain rule:

$z = F(x, y) = 6x + y^3$, $x = 2t^2$, $y =^3 3t^3$.

(d) Define Homogeneous and Homothetic functions of two variables. Check for the homogeneity or homotheticity of the following:

(i) $(x, y) = \dfrac{\sqrt{x} + \sqrt{y}}{x + y}$

(ii) $z = \ln e^{\ln \left(x^2 + xy \right)}$

(iii) $z = \ln\left(\dfrac{x^2}{y^2} \right)$

(iv) $Z = \ln x^2 + \ln xy$

(e) Find the equation of the tangent plane at a = (1, 1, 5) to the graph of $f(x, y) = x^2 + 2xy + 2y^2$

(f) If $\mu=\sqrt{xy}$ is the utility function, the prices of both x and y are Rs. 10, and the money income is Rs. 100. Find the demands that maximize utility. Check the second order condition for maxima.