METRIC SPACES

MULTIPLE CHOICE QUESTIONS-

1) If d is a mapping defined on metric space X, then, for every x,y∈ X :

   a)d(x, y) =0
   b)d(x, y) >=0
   c)d(x, y) <0
   d)None

   Ans: (b)

2)The metric space on R defined by d(x, y) =|x-y| is known as :
   a)Usual or standard metric
   b)Discrete metric
   c)Maximum metric
   d)None

   Ans :(a)

3)In a discrete metric space, for x=y, d(x, y) is defines as?
   a)1
   b)0

c)2
d)None
Ans:(b)

4)In a pseudometric defined on a non empty set X, x=y implies?
a)d(x, y) =0
b)d(x, y) >0
c)d(x, y) <0
d)None
Ans : (a)

5)In any metric space (X, d), X is?
a)Open
b)Closed
c)Both open and closed
d)None

Ans: (a)

6)The intersection of any finite collection of open sets is?
a)Open
b)Closed
c)Both open and closes
d)None
Ans: (a)

7)The intersection of infinite collection of open sets is?

a) Always open
b)Always closed
c)Need not be open
d)None

Ans: (c)

8)Every open ball is?

a)Open set
b)Closed set
c)Both open and closed
d)None

Ans: (a)

9)The non empty set (phi) is :

a) Open
b)Closed
c)Both open and closed
d)None

Ans: (c)

10) The Union of any finite collection of closed sets is :

a)Closed
b)Open
c)Both open and closed
d)None

Ans:(a)

11) Let A=[-2, 2] and X= set of real numbers, then diameter of (A), ie, d(A) is?

a)4
b)2
c)0
d)None

Ans: (a)

12) Every isometryis ?

a) One-one
b)Onto
c)Bijective
d)None

Ans: a)

13) A convergent sequence in a metric space is:

a) Not a cauchy sequence
b)Need not be a cauchy sequence
c)Cauchy sequence
d)None

Ans: (c)

14) If a subset A of metric space (X, d) contains each of it's limit points, the F is:
a)Open
b)Closed
c)Both open and closed
d)None

Ans : (b)

15) A metric space (X, d) is said to be complete if every cauchy sequence in X is:
a)Divergent
b)Convergent
c)Oscillatory
d)None

Ans : (b)

SUBJECTIVE-QUESTIONS

1.Prove that |(0, 1)| = |R|.

ANS:

Define $f : (0, 1) \to R$ by

$f(x) = 1/(x-0.5 + 2), x < 0.5$

$1/(x-0.5 - 2), x > 0.5$

$0, x = 0.5$

Note that the image of (0, 0.5) under f is $(-\infty, 0)$, the image of (0.5, 1) under f is $(0, \infty)$, and f(0) = 0. This shows that f is surjective. Furthermore, it is easy to check that f is injective when restricted to (0, 0.5) or (0.5, 1). As the images of these intervals are disjoint from each other and 0, it follows that f is injective on (0, 1). So f is a bijection as desired.

2. Check whether (X,d) is metric or not

ANS:

Let X be any non emptyset .

Define $d : X \times X \to R$ as

d(x, y) = 0 x = y

1 x is not equal to y

We now show that the following

(R, d) defines a metric.

$d(x, y) > 0 \ \forall x, y \in X$ $d(x, y) = 0$

$\Leftrightarrow x = y$ $d(x, y) = d(y, x) \ \forall x, y \in X$

All of these follow from the definition. Now we show the triangular inequality.

Let $x, y, z \in X$

To show $d(x, y) \leq d(x, z) + d(z, y)$

Note: LHS = 0 or 1 ,

whereas RHS = 0, 1, 2

Case(i) RHS = 2.

Then, trivially L.H.S < R.H.S.

Case(ii) RHS = 1 Then, LHS≤RHS Case

(iii) RHS=0 Then $d(x, z) = d(z, y) = 0$ [∵ each is non-meg]

∴ x = z and z = y∴ x = y∴ LHS = 0 so that LHS = RHS.

So, we see that in all the three cases we have LHS≤RHS. This metric is called discrete metric.

3) Prove the below theorem

ANS:

A convergent sequence in a metric space is a Cauchy sequence

Let $\{x_n\}$ be a sequence in a set X with metric d,

and let x be an element of X . Given any $\varepsilon > 0$,

there exists some natural number n_0 such that

$d(x_n, x) < \varepsilon/2$ whenever $n \geq n_0$. Consider any natural numbers n and m such that $n \geq n_0$ and $m \geq n_0$

Then $d(x_n, x) < \varepsilon/2$

and $d(x_m, x) < \varepsilon/2$. Therefore $d(x_n, x_m) \leq d(x_n, x) + d(x_m, x) < \varepsilon/2 + \varepsilon/2 = \varepsilon$

hence proved

4) Define Cauchy sequence in metric space.

ANS:

Cauchy Sequence Definition -.

Let d be a metric on a set X.

A sequence $\{x_n\}_n \geq 1$ in the set X is said to be a Cauchy sequence if, for every $\varepsilon > 0$, there exists a natural number n0 such that $d(x_n, x_m) < \varepsilon$ whenever $n \geq n_0$ and $m \geq n_0$

5) Define Convergence in case of a usual metric.

ANS:

Let d be a metric on a set X and

$\{x_n\}$ be a sequence in the set X. An element $x \in X$ is said to be a limit of $\{ x_n \}$ if, for every $\varepsilon > 0$, there exists a natural number n_0 such that $d(x_n, x) < \varepsilon$ whenever $n \geq n_0$

6) Define divergence in metric space.

ANS:

In this case, we also say that $\{x_n\}$ converges to x, and write it in symbols as $x_n \to x$.

If there is no such x, we say that the sequence diverges. A sequence is said to be convergent if it converges to some limit, and divergent otherwise.

7) Define sequence in metric space

ANS:

Let (X, d) be a metric space.

A sequence of points in X is a function f from N into X.

In other words, a sequence assigns to each $n \in N$ a uniquely determined element of X. If $f(n) = x_n$, it is customary to denote the sequence by the symbol $\{ x_n \}_{n\geq1}$

or $\{x_n\}$ or by $x_1, \ldots, x_n, \ldots$

8) When a metric space is called convergent?

ANS:

A metric space (X, d) is said to be complete if every Cauchy sequence in X is convergent.

9 How will you define isometry?

ANS:

A homeomorphism, also called a continuous transformation, is an equivalence relation and one-to-one correspondence between points in two geometric figures or topological spaces that is continuous in both directions.
A homeomorphism which also preserves distances is called an isometry.

10) How will you define homeomorphism?

ANS:

A homeomorphism, also called a continuous transformation, is an equivalence relation and one-to-one correspondence between points in two geometric figures or topological spaces that is continuous in both directions.

- (1) *h* is a one-to-one correspondence between the elements of *X* and *Y*;
- (2) *h* is continuous: nearby points of *X* are mapped to nearby points of *Y* and distant points of *X* are mapped to distant points of *Y*—in other words, “neighbourhood” are preserved;
- (3) there exists a continuous **inverse function** h^{-1}

11)How will you define continuity incase of metric spaces

ANS:

A function *f* mapping a topological space *X* into a topological space *Y* is defined to be continuous if, for each open set *V* of *Y*, the subset of *X* consisting of all points *p* for which *f*(*p*) belongs to *V* is an open set of *X*.

12) DefineSequential Criterion for the Continuity of a Function on Metric Spaces

ANS:

Let (S,d_S) and (T,d_T) be metric spaces and

let $f:S\rightarrow T$. Then f is continuous at the point $p\in S$ if and only if for all sequences $(x_n)^{\text{infinity}}{}_{n-1}=1$ in S that converge to p we have that $(f(x_n))^{\infty}{}_{n}=1$ in T converges to $f(p)$.

13) Define subspaces in metric spaces.

A metric subspace (A, dA) of (X, d) consists of a subset A ⊂ X whose metric dA : A × A → R is is the restriction of d to A; that is, dA(x, y) = d(x, y) for all x, y ∈ A. We can often formulate intrinsic properties of a subset A ⊂ X of a metric space X in terms of properties of the corresponding metric subspace (A, dA).

14). When a metric space is called seperable?

ANS:

A topological space is called separable if it contains a countable, dense subset; that is, there exists a sequence of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

15)Define sequential characterization of limit point in metric space

ANS:

In a metric space, sequential limits are unique.A sequence $\{x_n\}$ converges in a metric space (X,d) to a point x_0 is equivalent to the condition that for each ε> 0 there is a natural number N such that N >n implies $d(x_n , x_0) < ε$. For al;N>n . The sequence $f_n(x) = x^n$ belongs to C[0,1] but does not converge.