DSE II

DISCRETE MATHEMATICS

Subjective Questions

Ques-1 At most social functions there is a lot of handshaking. Prove that the number of people who shake hands with an odd number of people is even.

Ans-1) If we represent this problem by a graph in which vertices correspond to the people and an edge between two vertices means that those two people shook hands,then the number of hands a person shook is the degree of the corresponding vertex.

Then the result follows from the fact that number of odd vertices ina pseudograph is always even.

Ques-2 For given the sequence,either draw a graph whose degree sequence is as specified or explain why no such graph exists?

4,4,4,3,2

Ans-2) No such graph exists as sum of degrees of all vertices is 17 which is odd.

Ques-3 For given the sequence,either draw a graph whose degree sequence is as specified or explain why no such graph exists?

Ans-3) No such graph exists as sum of degrees of all vertices is 17 which is odd.

Ques-4) For given the sequence,either draw a graph whose degree sequence is as specified or explain why no such graph exists?

(100,99,98…..,3,2,2,2)

Ans-4) No such graph exists as sum of degrees of all vertices is 17 which is odd.

Ques-5) For given the sequence,either draw a graph whose degree sequence is as specified or explain why no such graph exists?

(5,5,4,3,2,1)

Ans-5)Such a graph doesn't exist.

Reason: Two vertices have degree 5 implies each of the other four vertices must have degree atleast 2 as a vertex of degree 5 in a graph of 6 vertices must be adjacent to each of the other vertices.

Ques-6) Show a$\leq b, iff\ a' \geq b'$.

Ans-6) $a \leq b\ implies\ \ a \vee b = b$

$= (a \vee b)' = b'$

$= a' \wedge b' = b'$

$$= a' \geq b'$$

Ques-7) Why a graph that contains a triangle cannot be a bipartite graph?

Ans-7) Atleast two of the vertices of a triangle must lie in one of the bipartition sets.Since those two are gained by an edge, the graph cannot be bipartite.

Ques-8) Is {1,2,3,6,9,18} a Boolean algebra under division?

Ans-8) It is not a Boolean algebra as 3 doesn't have a compliment.

Reason: $There\ does\ not\ exist\ y \in B, s.t\ 3 \vee y = 18\ and\ 3 \wedge y = 1$.

Ques-9) Let B be set of all divisors of 110. Show that (B,lcm,gcd) is a Boolean algebra?

Ans-9) 110 = 2X5X11 ; B = {1,2,5,10,11,22,55,110} ; IBI = 8 ; Clearly (B,lcm,gcd) is a lattice and B is Distributive. 'zero element' = 1 ; 'unity element' =

110. Every element has a complement. Therefore, (B,lcm,gcd) is a boolean algebra.

Ques-10) Give an example of poset with one maximal element but no greatest element.

Sol-10) Let P= $\{2^n : n \in \mathrm{N} \cup \quad \{3\}$ under divisibility

Then 3 is a maximal element,but there is no greatest element of P.

Ques-11) What is the significance of the total no. of 1's in the adjacency matrix of a graph?

Ans-11) Each 1 in the adjacency matrix represents an edge. Every edge $v_i v_j$contributes two 1's to the adjacency matrix.Thus number of 1's in the adjacency matrix is twice the number of edges.

Ques-12) Prove that any two Boolean algebras with same finite cordinality are isomorphic.

Ans-12) LetB_1 and B_2be the two finite Boolean algebras with same finite cordinality.

Let A_1 and A_2denote the set of atoms of B_1 and B_2 respectively.

Let O$(A_1) = n$ and $O(A_2) = m,$

Then $B_1 \cong P(A_1) \cong \{0,1\}^n = O(B_1) = 2^n$

$B_2 \cong P(A_2) \cong \{0,1\}^m = O(B_1) = 2^m$

But $O(B_1) = O(B_2)\ implies\ n = m$

Therefore ,$B_1 \cong \{0,1\}^n = B \quad _2$.

Ques-13) Explain why any graph is isomorphic to a subgraph of some complete graph.

Ans-13) It clearly follows from the fact that any graph G with n vertices in a subgraph of Kn.

Since we can obtain Kn from G by joining all pairs of vertices in G where there is not an edge already. Thus the results become valid.

Ques-14) Prove that if a lattice is isomorphic to a sublattice of a product of distributive lattices,then it is distributive.

Ans-14) Let L$\approx S\ where\ S\ is\ a\ sublattice\ of\ M \times K;$

Where both M and K are modular,

As M and K are modular,

M$\times$
$K\ is\ modular\ \ (cordinal\ product\ of\ modular\ is\ modular)$

S is modular (Sublattice of modular is modular)

L is modular (homomorphic image of modular is modular).

Ques-15) Derive the idempotent laws from the absorption laws only.

Ans-15) Absorption law: a∨ $(a \wedge b) = a \ \forall \ a, b \in <$

In particular, taking a=b gives,

a∨ $(a \wedge b) = a \forall \ a \in <$

therefore, a∧ $[a \vee (a \wedge a) = a \wedge a \ \ \forall \ a \in <$

i.e a= $a \wedge a \forall \ a \in <$

Similarly, a=a∨ $a \ \ \forall \ a \in <$.

<u>Multiple Choice Questions</u>

<u>Q-1)</u> If

$G_1\ and\ G_2\ are\ iso, orphic\ graphs, then\ G_1\ and\ G_2\ have$:

1)Same no. of vertices
2)Same no. of edges
3)Same degree of sequence

Which of the following is true?

a)Only 1

b)2 and 3
c)1 and 3
d)Only 3
e)1,2 and 3

Ans-1) e

Ques-2) The rule f$\sim g \; iff \; f(b_1, b_2, b_3, \dots \dots . b_n) = g(b_1, b_2, b_3, \dots \dots, b_n) \; is \; valid \; for \; all$

a)Boolean polynomials

b)Modular lattices

c) Binary n-tuples

d)Linearly ordered sets

Ans-2) c

Ques-3) Which of the following gate symbolizes pierce operation, i.e, (x1+x2)'.

a)Sub-Junction-Gate
b)NAND-Gate
c)NDT Gate
d)NOR Gate

Ans-3) d

Ques-4) Choose the correct option

a)Every modular lattice is distributive
b)Any chain is distributive
c)Any powerset lattice is not distributive
d)None of the above

Ans-4) b

Ques-5) Let G be a group and n be the class of all normal subgroups of G, then(n,≤) is a

a)Lattice
b)Poset
c)Modular lattice
d)All of the above

Ans-d

Ques-6) The complement of every element is unique

a)True
b)False
c)Inadequate data
d)Complement cant be determined

Ans-6) b

Ques-7) In graph theory, an edge that is incident with only one vertex is called a/an

a)Isolator

b)Loop
c)Pseudograph
d)Subgraph

Ans-7) b

Ques-8) If f is a Boolean isomorphism,then f is also said to be

a)Boolean homomorphism
b)Bijective
c)Complement preserving
d)All of the above

Ans-8) d

Ques-9) Thecordinality of finite Boolean algebra is always of the form

a)N

b) N!

c) (N-1)/2

d) 2^N

Ans-9) d

Ques-10) The conjunctive normal form of p= $x_1x_2' + x_1'x_2 \; can \; be$

a)$x_1 + x_2$
b)$x_1' + x_2'$
c)$(x_1 + x_2)(x_1' + x_2')$
d)$(x_1 - x_2)(x_1' - x_2')$

Ans-10) c

Ques-11) using the axioms of Boolean algebra,the Boolean polynomial xy+xy’+x’y can be simplified to

a)x+y
b)xy
c)x-y
d)x

Ans-11) a

Ques-12) Let P={2,3,5;12,18;30,40,45} is a poset under divisibility. Then {2,3,5} is

a)chain
b)quasi-order
c)anti chain
d)order homomorphism

Ans-12) c

Ques-13) A map that is order preserving not always be order embedding.

The given statement is

a)True
b)False
c)Valid only for R
d)None of the above

Ans-13) a

Ques-14) An infinite chain always has a top and bottom element

The above statement is

a)True
b)False
c)Inadequate data
d)None of the abOve

Ans-14) b

Ques-15) A relation $\leq$ on a set P is called partial order iff it is

a)Reflexive and transitive
b)Anti-symmetric
c)Both a and b
d)None of the above

Ans-15) c