PARTIAL DIFFERENTIAL EQUATIONS

MULTIPLE CHOICE QUESTIONS :-

Q1. A partial differential equation has :-

(A) one independent variable

(B) two or more independent variables

(C) more than one dependent variable

(D) equal number of dependent and independent variables

Solution - The correct answer is (B)

Q2.The partial differential equation 3. The partial differential equation

$$5\partial^2 z / \partial x^2 + 6\partial^2 z / \partial y^2 = xy$$ is classified as

(A) elliptic

(B) parabolic

(C) hyperbolic

(D) none of the above

Solution - The correct answer is (A).

Q3. The partial differential equation $xy \, \partial z/\partial x = 5 \, \partial^2 z/\partial y^2$ is classified as-

(A) elliptic

(B) parabolic

(C) hyperbolic

(D) none of the above

Solution - The correct answer is (B)

Q4. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation.

$$\partial w/\partial t + \partial^3 w/\partial x^3 - 6w \, \partial w/\partial x = 0$$

(A) linear; 3rd order

(B) nonlinear; 3rd order

(C) linear; 1st order

(D) nonlinear; 1st order

Solution - The correct answer is (B).

The partial differential equation is nonlinear because the coefficient of the derivative term $\partial w/\partial x$ is a function of the dependent variable, w. The equation is a 3rd order as that is the highest derivative in the partial differential equation.

Q5. Which of these equations are used to classify PDEs?

(A) b(dydx)−c=0
(B) a(dydx)2−b(dydx)=0
(C) (dydx)2−(dydx)+1=0
(D) a(dydx)2−b(dydx)+c=0

Solution – The correct answer is (D)

Q6. When solving a 1-Dimensional wave equation using variable separable method, we get the solution if ______________

(A) k is positive
(B) k is negative
(C) k is 0
(D) k can be anything

Solution – The correct answer is(B)

Q7. When solving a 1-Dimensional heat equation using a variable separable method, we get the solution if _______________

(A) k is positive
(B) k is negative

(C) k is 0

(D) k can be anything

Solution – The correct answer is(B)

Q8. Find the solution of $\partial u\partial x=36\partial u\partial t+10u$ if $\partial u\partial x(t=0)=3e-2x$ using the method of separation of variables.

(A) $-32e-2xe-t/3$

(B) $3exe-t/3$

(C) $32e2xe-t/3$

(D) $3e-xe-t/3$

Solution – The correct answer is(A)

Q9. Solve $\partial u\partial x=6\partial u\partial t+u$ using the method of separation of variables if $u(x,0) = 10\ e^{-x}$.

(A) $10\ e^{-x}\ e^{-t/3}$

(B) $10\ e^{x}\ e^{-t/3}$

(C) $10\ e^{x/3}\ e^{-t}$

(D) $10\ e^{-x/3}\ e^{-t}$

Solution – The correct answer is(A)

Q10. While solving a partial differential equation using a variable separable method, we equate the ratio to a constant which?

(A) can be positive or negative integer or zero

(B) can be positive or negative rational number or zero

(C) must be a positive integer

(D) must be a negative integer

Solution – The correct answer is(B)

SUBJECTIVE QUESTIONS -

Q1. What is partial differential equation?

ANS- A differential equation that contains one or more partial derivatives of the dependent variable is called the partial differential equation.

Q2. What is the general form of a partial differential equation?

ANS- The general form of a partial differential equation is –

$$F(x, y, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, u, u_x, u_y, \_\_\_\_\_\_\_, u_{xx}, u_{xy}, u_{yy}) = 0$$

Q3. How does initial and boundary condition helps?

ANS- The initial and boundary condition helps us to choose a particular solution out of the many solutions.

Q4. What is characteristic curve?

ANS- A curve in (x , y , u), whose tangent at every point coincides with the characteristics direction field is called the characteristic curve.

Q5. Find the solution of u_x-u_y= 1 , with the Cauchy data u(x , 0) = x^2 .

ANS- characteristic equation :-

dx/1 = dy/(-1) = du/1

dx = -dy

x + y = c1 -----------equ(i)

now,

dx = du

x + c2 = u -----------equ(ii)

byequ(i) and equ(ii)-----

u – x = c2 = f(c1) = f(x+y)

GENERAL SOLUTION:-

u – x = f(x+y)

$u(x, y) = x + f(x + y)$

$x^2 = u(x,0) = x+f(x)$

$x^2 = x + f(x)$

$f(x) = x^2 - x$

$f(x + y) = (x + y)^2 - (x+y)$

$u = x + (x + y)^2 - (x + y)$

NOW PARTICULAR SOLUTION :-

$u(x, y) = x+ (x + y)^2 - (x + y)$

$u(x, y) = (x + y)^2 - y$

Q6. Find the nature of the one-dimensional heat equation.

$\partial T / \partial t = \alpha\, \partial^2 T / \partial x^2$

Solution : - The general equation is in this form.
$A\partial 2\Phi\partial x2+B\partial 2\Phi\partial x\partial y+C\partial 2\Phi\partial y2+D\partial\Phi\partial x+E\partial\Phi\partial y+F\Phi+G=0$
Comparing $\alpha\partial 2T\partial x2-\partial T\partial t=0$ with the above equation, (let 'y' be 't').
$A=\alpha$
$B=0$
$C=0$

To find the type,
$d=B^2-4AC$
$d=0$
As d is zero, the one-dimensional heat equation is parabolic.

Q7. What is the form of Euler equation?

ANS- $Au_{xx} + Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$

Q8. Reduce $yu_x + u_y = x$ to canonical form and hence find the general solution.

ANS- $yu_x + u_y = x$ -------equ(i)

$dx/y = dy/1 = du/x$

$dx = ydy$

$\xi = x = y^2 + c1$ --------equ(ii)

we choose –

$\eta(x, y) = y = c2$

$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi * 1 + u_\eta * 0 = u_\xi$

$u_y = u_\xi \xi_y + u_\eta \eta_y = u_\xi*(-y) + u_\eta * 1 = (-y) * u_\xi + u_\eta$

so, equ (i) reduces to –

$yu_\xi - yu_\xi + u_\eta = x$

$u_\eta = x$

$u_\eta = \xi + \eta^2 /2$ ---------equ(iii)

fromequ(ii)-

$x = \xi + y^2/2$

$x = \xi + \eta^2/2$

INTEGRATING equ(iii)---

$u(\xi , \eta) = \xi\eta + 1/2 *(n)^3/3 + f(\xi) = \xi\eta + 1/6 * \eta^3 + f(\xi)$

so, equ (i) in terms of x and y will be-

$u(x,y) = xy – 1/6 * y^3 +f(x – y^2/2)$

Q9. What is the necessary condition for obtaining the 2nd derivative in Cauchy problems?

ANS- The necessary condition for obtaining the 2nd derivative is that the curve Lo must not be a characteristic curve.

Q10.Explain how PDE are formed?

Ans- PDE can be obtained

(i) By eliminating the arbitrary constants that occur in the functional relation between the dependent and independent variables.

(ii) By eliminating arbitrary functions from a given relation between the dependent and independent variables.

Q11.Mention three types of solution of a p.d.e (or) Define general and complete integrals of a p.d.e.

Ans-

(i) A solution which contains as many arbitrary constants as there are independent variables is called a complete integral (or) complete solution.

(ii) A solution obtained by giving particular values to the arbitrary constants in a complete integral is called a particular integral (or) particular solution.

(iii) A solution of a p.d.e which contains the maximum possible number of arbitrary functions is called a general integral (or) general solution.

Q12.Form the partial differential equation by eliminating the arbitrary constants a and b from

$z = (x2 +a2) (y2 + b 2)$

Ans- Given $z = (x2 + a2)(y2 + b2)$ ……..(1)

Differentiating (1) partially w.r.t x &y , we get

$p = 2x \ (y2 + b2)$

$q = 2y \ (x + a)$

Substituting the values of p and q in (1), we get

$4xyz = pq$

which is the required partial differential equation.

Q13.Find the partial differential equation of the family of spheres of radius one whose centre lie in the xy - plane.

Ans- The equation of the sphere is given by

$(x - a)^2 + (y - b)^2 + z^2 = 1$ ______ (1)

Differentiating (1) partially w.r.t x &y , we get

$2(x-a) + 2zp = 0$

$2(y-b) + 2zq = 0$

From these equations we obtain

$x-a = -zp$ ___ (2)

$y-b = -zq$ ___ (3)

Using (2) and (3) in (1), we get

Z^2p^2 + z^2q^2 + z^2 = 1

or z^2 (p^2 + q62 + 1) = 1

Q14.Form the partial differential equation by eliminating the arbitrary function f from

z = ey f (x + y)

Ans-

Consider z = ey f (x +y) ______ (1)

Differentiating (1) partially w .r. t x & y, we get

p =ey f ' (x + y)

q =ey f '(x + y) + f(x + y). ey

Hence, we have

q = p + z

Q15. Solve q = yp2

Ans- This is of the form f (y,p,q) = 0

Then, put p = a.

Therfore, the given equation becomes q = a2y.

Since dz = pdx + qdy, we have

$dz = adx + a2y\, dy$

Integrating, we get $z = ax + (a2y2/2) + b$

Q16. Solve $z = px + qy + pq$

Ans- The given equation is in Clairaut‟s. form

Putting $p = a$ and $q = b$, we have

$z = ax + by + ab$ -------- (1)

which is the complete integral.

To find the singular integral, differentiating (1) partially w.r.t a and b, we get

$0 = x + b$

$0 = y + a$

Therefore we have, $a = -y$ and $b= -x$.

Substituting the values of a & b in (1), we get

$z = -xy - xy + xy$

or $z + xy = 0$, which is the singular integral.

To get the general integral, put $b = F(a)$ in (1).

Then $z = ax + F(a)y + a F(a)$ ---------- (2)

Differentiating (2) partially w.r.t a, we have

0 = x + F'(a) y + aF'(a) + F(a) ---------- (3)

Eliminating „a‟ between (2) and (3), we get the general integral.

Q17.While solving any partial differentiation equation using a variable separable method which is of order 1 or 2, we use the formula of fourier series to find the coefficients at last.

a) True

b) False

Answer: TRUE

Explanation: After using the boundary conditions, when we are left with only one constant and one boundary condition, then we use Fourier series coefficient formula to find the constant.

Q18.While solving a partial differential equation using a variable separable method, we assume that the function can be written as the product of two functions which depend on one variable only.

a) True

b) False

Answer: a

Explanation: If we have a function u(x,t), then the function u depends on both x and t. For using the variable separable method we assume that it can be written as u(x,t) = X(x).T(t) where X depends only on x and T depends only on t.

Q19.What are the lines along which the derivatives of the dependent variables are indeterminate called?

Answer: The characteristic lines determine the type of the equation. These are defined as the lines along which the derivatives of the dependent variables do not exist.

Q20.What is the classification of PDEs governed by?

ANS- The classification of PDEs are governed by their highest order derivatives.