DSE-I

NUMERICAL ANALYSIS

1. We wish to solve $x^2 - 2 = 0$ by Newton Raphson technique. If initial guess is $x_0 = 1.0$, then find subsequent estimate of x (i.e. x_1) ?

Solution: We know that iterative scheme for newton method is given by

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$x_0 = 1.0$ (given) then we need to find x_1

so using the above formula

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 + \frac{1}{2}$$

$$x_1 = \frac{3}{2}$$

2. Find the interval in which the roots of interval lie f(x) = $x^3 - 4x + 9$

Solution: We will use Intermediate value theorem to find the intervals in which roots of the equation lies.

f(0) = 9 > 0 and

f(-3) = -6 < 0

therefore, root lie in the interval (-3,0).

3. When does Newton method fails to provide us the root?

Solution : Newton method fails to provide us the root when tangent to the curve is parallel to x –axis as f'(x)=0

4. Which sequence will converge with faster rate of convergence

$\frac{n+3}{n+7} or \frac{2^n+3}{2^n+7}$?

Solution: The sequence $\frac{2^n+3}{2^n+7}$ will converge with faster rate as compared to $\frac{n+3}{n+7}$.

This is because $\frac{1}{2^n}$ ->0 faster as compared to $\frac{1}{n}$.

5. What are the disadvantages of false position method?

Solution: a) Rate of convergence is very slow.

b) Good intermediate approximations may be discarded .As a result it is possible that an

approximation that is accurate to within specified convergence will fail to terminate the

iteration.

6. What is the major difference between newton and secant method?

Solution: The major difference is that newton method uses a line that is tangent to one point, while the secant method uses a line that is secant to two points.

7. What is the computational cost of LU decomposition of n*n matrix ?

Solution: O($\frac{n^3}{3}$)

8. If A=LU ,then value of det(A)?

Solution: Det(A) = $(l_{11} * l_{22} * \dots * l_{nn}) * (u_{11} * u_{22} * \dots * u_{nn})$

9. Convert the given matrix in form of LU decomposition

A=$\begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix}$

Solution: We have ,

A = LU

L=$\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}$ and U = $\begin{bmatrix} u_{11} & u_{21} \\ 0 & u_{22} \end{bmatrix}$

$$\begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{21} \\ 0 & u_{22} \end{bmatrix}$$

This implies $l_{21} = 2, u_{11} = 1, u_{21} = 5, u_{22} = -9$

Thus, A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & -9 \end{bmatrix}$

10. What is the condition for convergence of Gauss Siedel method?

Solution: The Gauss- Seidal method converges if the number of roots inside the unit circle is equal to the order of the iteration matrix.

11. Does Newton method always converge? Give example.

Solution: No, newton method do not converge always. It depend upon initial approximation for root. Consider f(x) = tan(πx) – x – 6.If we choose initial approximation

as 0.48 ,we get root for the equation .But when choose intial approximation as 0.4 then

newton method fails to converge even after 5000 iterations.

12. Which method is also known as “method of simultaneous relaxation”?

Solution: Gauss – Jacobi method is also known as “method of simultaneous relaxation”.

13. How many function evaluation occur in bisection and newton method in one iteration?

Solution: Bisection method : 1

Newton method : 2

14. Perform 1 iteration to find root of the function f(x) = $\frac{1}{x} - 37$ in (0.01,0.1) using false position method?

Solution: The iterative scheme formula is given by

$$p_n = b_n - \frac{f(b_n)(b_n - a_n)}{f(b_n) - f(a_n)}$$

f(0.1) = -27 , f(0.01) = 63

Using this iterative formula we get

$p_1 = 0.073$

15. Does root exist for f(x) =$x^3 - 2x^2 - 3x + 1$ in the interval (0,1)?

Solution: Yes , the root exist in interval (0,1) because

f(0) = 1> 0 and f(1) = -3 < 0

Thus, by intermediate value theorem there exist a root in interval (0,1)

16. Numerical solutions of linear algebraic equations can be obtained by

A. EULER'S MODIFIED METHOD
B. RANGA KUTTA METHOD
C. EULER'S MODIFIED METHOD
D. NONE OF THESE

Answer- C)

17. THE SMALLEST +VE ROOT OF X^3-5X+3=0 LIES BETWEEN

A. 0 AND 1
B. 2 AND 3
C. 1 AND 2
D. NONE OF THESE

ANSWER – C)

18. THE NEWTON RAPHSON METHOD IS ALSO CALLED AS _____

A. TANGENT METHOD
B. SECANT METHOD
C. CHORD METHOD
D. DIAMETER METHOD

ANSWER- A

19. FOR DECREASING THE NUMBER OF ITERATIONS IN NEWTON RAPHSON METHOD:

A. THE VALUE OF F'(X) MUST BE INCREASED
B. THE VALUE OF F''(X) MUST BE DECREASED

C. THE VALUE OF F’(X) MUST BE DECREASED

D. THE VALUE OF F’’(X) MUST BE INCREASED

ANSWER- A)

20. IN THE GAUSS ELIMINATION METHOD FOR SOLVING A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS, TRIANGULARZATION LEADS TO

A. Diagonal matrix

B. Lower triangular matrix

C. Upper triangular matrix

D. Singular matrix

Answer – c)

21. MATCH THE FOLLOWING:

A. Newton-Raphson 1. Integration

B. Range-kutta 2. Root finding

C. Gauss-seidel 3. Ordinary Differential Equations

D. Simpson’s 4. Solution of system of Linear Equations

The correct sequence is

1. A2-B3-C4-D1
2. A3-B2-C1-D4
3. A1-B4-C2-D3
4. A4-B1-C2-D3

Answer – 1

22. THE NEWTON-RAPHSON METHOD FAILS IF IN THE NEIGHBORHOOD OF ROOT

A. F”(x)=0
B. F’(X)=0
C. F(x)=0
D. None of these

Answer – b)

23. The roots of equation $x^3-x-9=0$ near x= 2 correct to three decimal places by using Newton-Raphson method

A. 2.240
B. 2.242
C. 2.241
D. 2.273

Answer – b)

24. USING BISECTION METHOD , THE REAL ROOTS OF BETWEEN X=2 AND X=4 IS NEAR TO

A. 2.2
B. 2.75
C. 4.0
D. 2.5

Answer- b)

25. BISECTION METHOD IS ALSO KNOWN AS

A. Interval halving method
B. bolzano and Interval Halving method
C. Bolzano method
D. successive method

Answer- B

26. THE CONVERGENCE OF WHICH OF THE FOLLOWING METHOD IS SENSITIVE TO STARTING VALUE?

A. False position
B. Gauss seidal method
C. Newton- raphson method
D. All of these

Answer- c)

27. WHICH OF THE FOLLOWING STATEMENTS APPLIES TO THE BISECTION METHOD USED FOR FINDING ROOTS OF FUNCTIONS?

A. CONVERGES within a few iterations
B. GUARANTEED to work for all continuous functions
C. IS faster than the Newton-Raphson method
D. REQUIRES that there be no error in determining the sign of the function

Answer- B)

28. GIVEN N+1 DATA PAIRS, A UNIQUE POLYNOMIAL OF DEGREE _____ PASSES THROUGH THE N + 1 DATA POINTS.

A. n +1
B. n
C. n or less
D. n +1 or less

Answer – c)

29. LAGRANGE'S INTERPOLATION FORMULA IS USED TO COMPUTE THE VALUES FOR _____ INTERVALS.

A. EQUAL

B. UNEQUAL
C. OPEN
D. CLOSED

Answer- b)

30. AS SOON AS A NEW VALUE FOR A VARIABLE IS FOUND BY ITERATION, IT IS USED IMMEDIATELY IN THE FOLLOWING EQUATION. THIS METHOD IS CALLED_____.

A. GAUSS Seidel
B. GAUSS Elimination
C. GAUSS Jacobi
D. Gauss Jordan

Answer- A)